Logrolling under Fragmented Authoritarianism: Theory and Evidence from China

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ABSTRACT

Although political economy has achieved significantly progress in understanding authoratarian systems, the inner working of policy-making in authoratarian polities is still obscure. In particular, logrolling or trading of favors among vertical bureaucratic systems in China has been extremely under researched. An important progress in the analytical methodology has been provided by the "Fragmented Authoritarianism" framework developed by China study scholars to analyze the policy making problems in a vertically segmented political system. However, although this framework helps illuminate the policy process in China, it lacks formal modeling and a full econometric testing. This paper develops a formal model to study the effect of logrolling on policy making. We find that, policies under logrolling tend to be inefficiently high and policies excluded from logrolling tend to be inefficiently low. We provide empirical evidence to support our argument by studying the logrolling between Ministry of Civil Affairs and Ministry of Health. The consequence of logrolling is there is too high benefit associated with "Dibao" such that it even crowds out the unemployment insurance. Similarly, there is over enrollment in rural health insurance. However, there is insufficient input in infrastructure building in mental health care.

Key Words: Authoritarianism, Logrolling, Fragmented Authoritarianism, Policy Making, China.

> "The Chinese government makes policy according to a decision rule of delegation by consensus ..... If the agents reach consensus, the decision is automatically ratified by the higher level; if the agents cannot agree, then the authorities step in to make the decision, or the matter is dropped or tabled until consensus can be achieved".— Shirk 1993, p.116
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> "Instead of a small, stable group of central officials who are willing to trade votes on one issue because they trust others to pay them back on the next issue, there are larger, ad hoc groups that find logrolling and agreement much more problematic".— Shirk 1993, p.127

1. Introduction

In the past few decades, political economics has achieved quite a lot of progress towards analyzing the policy making process in democratic political systems. Nonetheless, the formation of policies in autocracies remains a difficult question for nowadays political economists. Conventional wisdom sees policy making in autocracy as a black-box where decisions are made in smoke-filled room. Therefore, there has been little attempt to build a general analytical framework on autocratic policy process. Based on the study of policy making in China, this paper tries to explain policy making in autocracy by developing an analytical framework with a specific focus on "logrolling" among groups have parochial interests, both theoretically and empirically. Simply speaking, logrolling means two (or more) interest groups trade their mutual support, so both can get benefit in the end. It is one way to reach consensus among the groups of roughly equal authority and have mutual veto power. An important thesis we intend to forward in this paper is that, because of logrolling, autocratic governments tend to adopt overreaching policies but lose the ability to pull back even when the costs outweigh its benefits. We will first study the bench-mark models where policies are made under dictatorship or bargaining separately, and then we will set up a simple logrolling model to compare the results with the bench-mark cases.

As an example to illustrate the concept of logrolling, consider the decision making process in nowadays China, which has long been treated as a consensus-making process, as the leaders reluctant to challenge one another. In China, bureaucratic interest groups are stove-piped - meaning they are separate vertical organizations, reaching down from Beijing to the provinces and cities. Each bureaucracy pushes for policies in their own interests - to increase its own budgets, staffing, etc.. The leaders make decisions by logrolling (or trading favors) with each other. They follow the rule, "I'll go along with what you want to do in your domain, if you let me call the shots in my domain"(Interview with Susan Shirk).

In this paper, we will illustrate the logrolling among two functional vertical organizations in China, say, the Ministry of Civil Affairs (MCA) and the Ministry of Health (MOH). Ministry of Civil Affairs(MCA) treats the "Minimum Livelihood Guarantee Scheme" (or "Dibao") initiated in urban areas as its core responsibility. MCA's objective is to provide as much financial and other assistances for households tagged as "Dibao" households as possible. The medical assistance, one important in-kind transfer determining the overall benefits of Dibao, is under the control of the MOH. MOH identifies who are

eligible to receive the medical assistance and sets the assistance standards[1]. Therefore, MCA wants the support of MOH on its agendas concerning the Dibao program, and has a strong incentive to trade favor with MOH.

On the other hand, rural health insurance ("New Cooperative Medical Scheme"), one of the most important health insurance programs in China, is the core interest of Ministry of Health. MOH's objective is to expand the coverage of rural health insurance, and the ideal position for MOH is to have universal coverage. However, a lot of poor households in the rural areas do not have the financial ability to pay for the insurance premium. They depend on the medical assistance fund controlled by MCA to help them cover the cost. Therefore, in order to increase the number of enrollees registered with the rural health insurance plan, MOH needs the cooperation of MCA on assistance concerning rural health insurance, and has a strong incentive to ask MCA to follow what MOH wants.

Logrolling in this context between MOH and MCA is that MCA uses its assistance fund to pay for the insurance premium for poor households in rural areas to register with rural health insurance plan. In exchange, MOH simply pegs the health care subsidy to "Dibao", i.e. the people who receive "Dibao" are automatically entitled to the subsidy of medical assistance.

The result of logrolling between MOH and MCA is over-reaching and inefficiency in both "Dibao" and rural health insurance. We find the overall benefit associated with "Dibao" is too high such that it even crowds out the unemployment insurance. Note that the cash benefit of Dibao (claimed from MCA) is not so high and does not crowd out unemployment insurance, but once other transfer-in-kind programs managed by other ministries (including medical assistance managed by MOH) are included. The overall benefits from Dibao may exceed the benefits offered from the unemployment insurance and consequently crowd out some unemployment insurance enrollees. Local officials who manage Dibao (under Ministry of civil affairs) set the line accordingly to make sure Dibao benefit itself does not crowd out unemployment insurance. However, officials in other ministries (including MOH) managing different social assistance programs simply use the "Dibao" as a tag to target the poor. In this case, overall benefits of Dibao including cash benefit (claimed from MCA managed "Dibao") and other in-kind-transfer (e.g. medical assistance) may surpass the benefits from unemployment insurance (author's interview).

Similarly, we find there is inefficiency for enrollees of rural health insurance in utilization of medical services. It is estimated that there are over 100 million people are being covered by more than one social health insurance programs in China.[2] People can only claim benefit from one of these social health insurance programs if they registered with more than one social health insurances. It is not efficient that enrollees pay premium and government pay subsidy for an insurance plan for which those enrollees may never claim benefit from.

Mental healthcare, which is one of the most important public health issues in China, is another policy arena for which both MOH and MCA are responsible. Both MOH and MCA are operating mental hospitals for treating patients. However, mental healthcare, which is not the priority issue of both MOH and MCA received deficiency in government investment. For example, regarding to psychiatric care, the current bed utilization rate

[1]See http://www.moh.gov.cn/mohzcfgs/s3577/200804/29695.shtml

[2]See, http://news.xinhuanet.com/politics/2014-08/15/c_126873413.htm

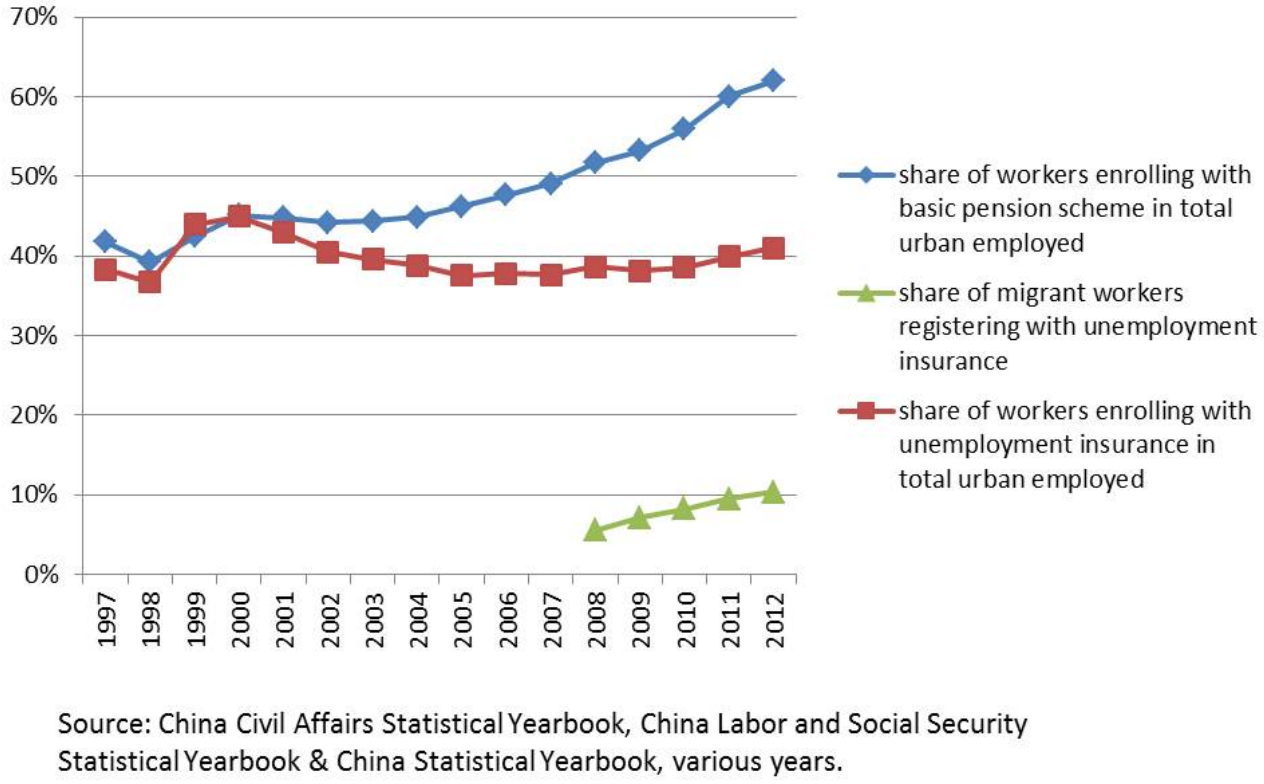

Figure 1. Share of enrollees of unemployment insurance and basic pension scheme in urban labor forces

is over 96% and average length of stay for inpatients is about 54 days in China, which is much longer than the national average of 8.9 days for all diseases and it is estimated that over 70% of patients with serious mental disease did not receive proper treatment in China (Qian 2012).

The following descriptive figures provide the overall glimpse of the inefficiencies caused by logrolling between MCA and MOH.

One stylized fact is that there are much more urban labor forces enrolling with another social insurance program — the Basic Pension Scheme (BPS) — than with the unemployment insurance (UI). From Figure 1, we can see there are only 40% of urban labor forces registered with unemployment insurance while over 60% of urban labor forces registered with the Basic Pension Scheme. Both BPS and UI are compulsory for urban labor forces and in principal the enrollment rate should be similar. However, the enrollment of UI is much lower, so we can infer that many urban labor forces chose to quit UI. And most of those who quit UI, join Dibao because of its higher benefit. From figure 2, we can see most of the people who enrolled in Dibao are actually unemployed or flexibly employed.

Figure 3 shows that the coverage of health insurance is universal. In 2013, the total

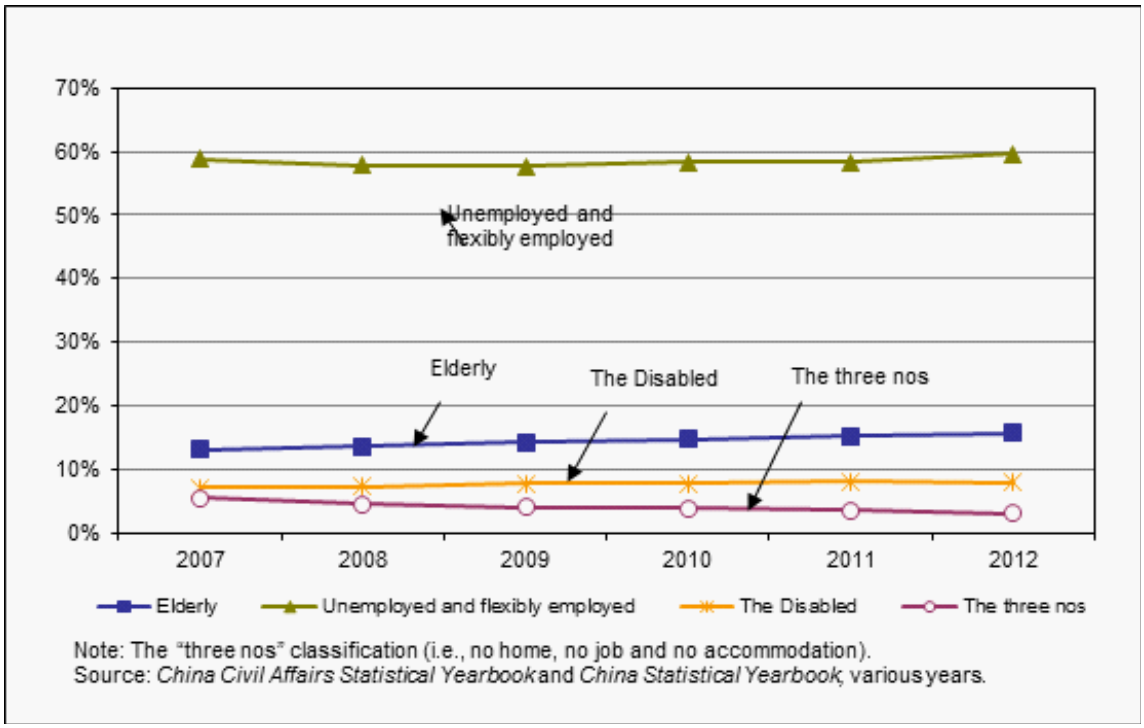

Figure 2. Composition of Beneficiaries under Dibao in urban areas

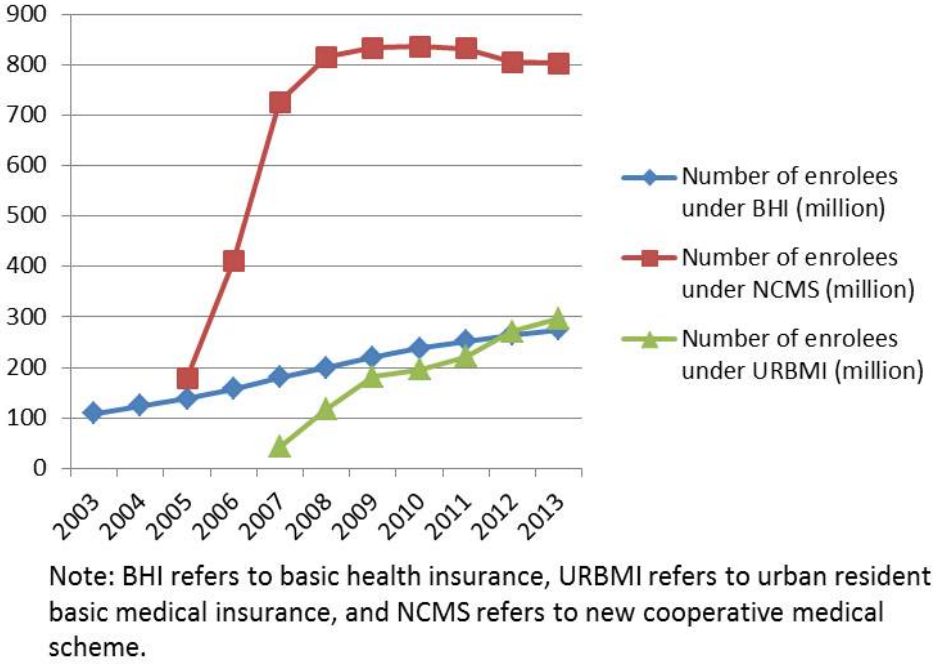

Figure 3. The number of enrollees under different social health insurances

number of enrolees under three social health insurance plans is over 1.37 billion which exceed the number of total population in China (1.36 billion, National Bureau of Statistics 2013). A large amount of people are covered by more than one social insurance plans. However, poor family may still have difficulties to pay for healthcare expenditure. The out-of-pocket expenditure is over RMB 1 trillion in 2013 (total health expenditure is about RMB 3 trillion).

Figure 4 shows that the share of government input in mental hospitals in total government health expenditure is decreasing in both types of hospitals under ministry of Health (MOH) and Ministry of Civil Affairs(MCA). World Health Organization (WHO)'s threshold level is at least 2% of total health expenditure should be allocated for mental healthcare, whereas the share of mental hospital expenditure in China is less than this threshold.

The paper proceeds as follows. Section 2 reviews the existing literature on logrolling and policy making in China in order to illustrate the contributions of our paper. Section 3 is a theoretically presentation of the analytical framework of logrolling and our finding. In section 4, we present the empirical model, the data and the empirical results. Conclusions are drawn in the final section.

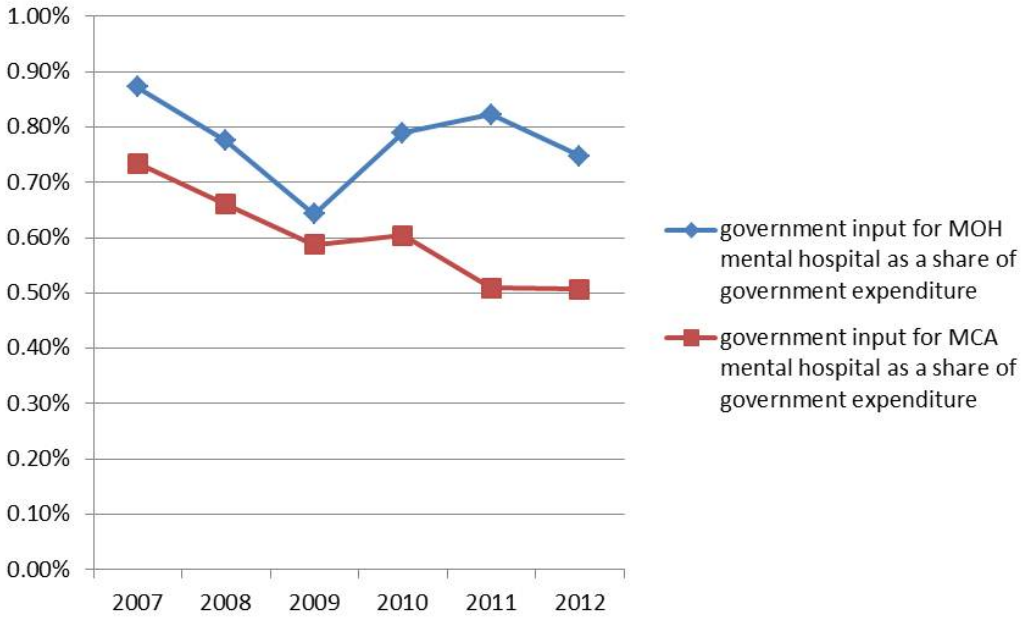

Figure 4. Government inputs in mental hospitals as a share of total government health expenditure

2. Literature Review

2.1. Literature on logrolling

Originated from the old custom in the lumber regions of Maine, where lumberjacks assisted one another in rolling the logs to the river after they were felled and trimmed (as in phrase you roll my log and I'll roll yours)[3], the word logrolling later was widely used in political science to refer to the phenomenon of trading of favors. The academic study of logrolling received substantial attentions since the discussion of the issue in The Calculus of Consent, in which logrolling is described as the process of vote-trading in democratic politics. "If the individual participant recognizes the economic value of his own vote to others on certain issues and, in turn, recognizes the economic value of others' votes to him on separate issues, he will be motivated to engage in 'trade'. ... The individual may effectively, but imperfectly, 'sell' his vote on a particular issue, securing in return the votes of other individuals on issues of more direct interest" (Buchanan and Tullock 1962, p. 92). To some degree, logrolling occurs in all political systems. The ruling coalition in Germany before the First World War was the nationalist "marriage of iron and rye", in which aristocratic landowners supported a fleet-building program that industrial interests desired; in exchange, big business supported high agriculture tariffs (Snyder 1991). In democracies, logrolling occurs in the form of vote trading by politicians, who trade support for one issue or piece of legislation in exchange for another politician's support to the issue of their own interest. For instance, a vote on behalf of a tariff of textile may be traded by a congressman for a vote from another congressman on behalf of a subsidy to the steel interests to ensure that both acts will gain a majority and pass through the legislature (Stratmann 1995). Being viewed as deviation from the orderly working of the democratic process, vote trading of this kind was believed would lead to too much government spending and socially inefficient policies even under majority rule (Tullock, 1959).

After the early studies of Buchanan and Tullock, the phenomenon of logrolling in policy making became a focus topic during the 1960s to 1970s, thereby economists and political scientist started to construct formal theories on it. For example, many thought logrolling would lead to inefficiency, emphasizing the welfare loss associated with vote trading and view trading votes as a negative-sum game (Wilson 1969; Riker and Brahms, 1973; Tullock 1970; Haefele 1971; Browning and Browning, 1979; Koford, 1982). But the efficiency issue was highly disputable as some argued that vote trading might be socially efficient, because it allowed legislators to express different intensities of preferences, making every legislator better off than he would have been in the absence of vote trading (Coleman 1966; Schwartz 1975). More recent works (Miller 1977; Enelow, 1986; Carrubba and Volden 2000) studied the conditions for the existence of logrolling arrangements under different voting rules. The limitation on most of the theoretical researches of logrolling is that they rely too heavily on spatial modeling approach which has been commonly used to study electoral competition or social choice, and to a large extend, ignoring the crucial strategic aspect of how agents interact in political and market environments. Focusing on

[3]http://www.etymonline.com/index.php?allowed_in_frame=0&search=logrolling&searchmode=none Online Etymology Dictionary, accessed on September 10, 2014. See also the film https://www.youtube.com/watch?v=H81BHC5pHb8.

the strategic interaction perspective, our paper takes a different approach from the existing literature. We study the logrolling process between different players by incorporating the rich literature on modern bargaining theory developed after the 1980s (see Rubinstein, 1982; Canning, 1990; Osborne and Rubinstein, 1990).

In its broad sense, logrolling can be treated as a special form of bargaining game. For example, in organizational analysis, logrolling is described as one of the ways of bargaining that negotiators can reach integrative agreements (Pruitt, 1981, 1983). Logrolling is a bargaining process that allows the parties to trade off their low-priority concerns to achieve high-priority concerns (ibdl.). But in its narrow sense, logrolling is different from the typical bargaining models described in the existing bargaining literature. In the classic bilateral bargaining (or Rubinstein bargaining) models (Rubinstein, 1982; Binmore 1986), a unique equilibrium exists where the first proposal is accepted when information is complete and the players are impatient. In the legislative bargaining models (Austen-Smith and Banks 1988; Baron and Ferejohn 1989; Alesina and Rosenthal 1996; Merlo 1997), the bargaining process is multilateral where members of legislature act non-cooperatively, and the bargaining outcomes is a function of rules of bargaining, the legislative structural, and the preference parameters (Banks and Duggan, 2000). Logrolling differs from these bargaining models in at least two aspects. First, both these bargaining models focus on the question of how to divide benefits among the members from one fixed pie, no matter the process is bilateral or multilateral. The main problem is that members have conflicting preference towards the allocation of benefits and there is no majority rule equilibrium in the standard social choice framework. However, these models do not involve the trading of favors among the members which is considered to be the essence of logrolling. In logrolling model considered here, there are at least two issues under consideration, and the players have different priorities on these issues so they can trade favors with each other. Second, in the Rubinstein bargaining and legislative bargaining literature, the agenda setter gets more benefits than the others in the sense that there exists first-mover advantage. By letting different players making proposals on different type of policies sequentially where the proposal power is symmetrically distributed, in the logrolling model considered here the first-mover advantage disappears. In order to see the differences in the policy outcomes, we separate the logrolling game from the bargaining game, and we compare the equilibrium outcomes under both games.

Another type of activities adopted by organized interest groups to promote their political objectives that have been widely discussed in the literature is lobbying. Logrolling is also different from lobbying activities. While lobbying takes place between interest groups and policy makers, logrolling takes place between parallel interest group. In the jargon of the economics literature, lobbying refers to meetings between representatives of interest groups and policymakers in which the former try to persuade the latter to promote the groups' objectives (Grossman and Helpman, 2001 p.104). Such lobbying activities may involve the transfer of information by verbal argument which is regard as "cheap talk", or the transfer of monetary contributions to policy maker (Persson, 1998; Grossman and Helpman, 2001). The common-agency model (Grossman and Helpman, 1994, 1995) has been widely used in the analysis of the allocation of group specific goods under lobbying activities. Generally, the organized groups get more than the social optimum, and the unorganized groups get less (Persson 1998; Persson and Tabellini, 2000).

A more important and relative question that has been under studied by the existing theoretical literature is whether logrolling will lead to overspending and overreaching in state policies. Through deep studies on the causes of the two world wars and the cold war, international relations scholar Jack Snyder warns logrolling by domestic interest groups might lead to the lack of constraint in military expansion. He points out that the danger of logrolling is some well-organized parochial interest groups can hijack national policy in directions that benefit themselves but are detrimental for the national good. Such policies may be inefficient and undercutting productivity which is also harmful to the ruling elite. But sometimes the state is unable to pull back from the detrimental policies even if the cost of over-reaching has exceeded its benefit to the ruling elite (Snyder 1991). Using a simple analytical framework, our paper shows that logrolling leads to overspending compared to the bench mark of benevolent social planner and bargaining. We find that through logrolling, projects that benefit particular interest groups can get the approval, even though the costs are dispersed throughout the entire populace. We may thus expect higher government spending on certain programs when logrolling is allowed to flourish, and the citizens seen as a "common pool" end up receiving less public goods than those who are not in a logroll system.

Most of the studies of logrolling are theoretical, because to testify the existence of logrolling depends on the record of trading of favors, which may not exist when the trade is implicit (Evans, 1994), or may difficult to acquire as such trade is usually considered morally reprehensible behavior (Buchanan and Tullock 1962). Stratmann (1992, 1995) made some progress in empirically test the phenomenon of logrolling in the US congressional voting in the 1960s and 1980s. Later, and Irwin and Kroszner (1996) provide evidence on how interest groups traded favors with each other in the passage of Smoot-Hawley Tariff Act of 1930 by calculating the votes. The most recent literature tries to apply the theory of logrolling to explain the equilibrium EU policies (Crombez 2000) and political economy of IMF lending (Copelovitch, 2010). However, in the context of Chinese politics, it is almost impossible to find such voting records. The existing empirical approach seems to be not feasible in this paper, due to lack of data. Instead, we take an indirect approach by first looking at the overreaching policy outcomes and then, use them to identify who are involved in logrolling and how they log roll with each other.

2.2. Literature on policy making in China

Making policy is the core function of all nation-states. Understanding the policy making process helps to open up the black box of China's domestic politics. During Mao Zedong's era, China has been treated as a near-totalitarian system (Richard Walker, 1955). Like all autocratic regimes, the ruler, not accountable to anyone else for what he does, makes decisions and enforces the implementation by using heavy doses of propaganda and coercion (Feridrich and Brzezinski, 1956). However, even under such a politically hierarchical system, in China economic decision-making involved a lot of negotiation and bargaining. Bargaining mainly took place between the superiors and subordinates, and concentrated on the level of commands coming down from above and the type of information going up. Those at lower levels, for example, have an interest in manipulating and distorting information in order to advance their own goals. It is observed in the literature that centrally planned economy in China was significant different from other planned economy

in that the Center's control over the economic system was not as strict as its control over the political system (Yingyi Qian and Xu 1993; Xu, 2011). Therefore, the decision making before the economic reform can be referred to as "command-bureaucratic" instead of "command", in order to capture both the authoritarian flavor and the sense of unresponsiveness due to the impoverished information flow (Naughton, 1992).

There have been dramatic changes in China's polity in the post-Mao reform era. The Chinese leaders, who have become progressively less dominant, have transformed toward more constrained figures who are primus inter pares within a collective group (Lampton, 2014, p.59). The top leaders need the support of the selectorate to remain in power (Shirk 1993). There was an increase in the size and power of the selectorate after Deng's reform, which created selectorate accountability in the political system (Gilli and Li, 2013). When the policy making system was dominated by strong leaders, the system had low friction, as subordinates did what they were told. But when the leaders became less dominant, the process of making policy became more complex and arduous.

Beginning from the end of the 1980s, a group of influential China Study scholars developed the "Fragmented Authoritarianism" framework[4]. In their view, the authority below the very peak of the Chinese political system is vertically fragmented (stove-piped), reaching down from Beijing to various levels near the bottom. These separate functional vertical organizations, such as various ministries, have equal rank according to China's bureaucratic ranking. Therefore, they cannot command each other. Besides, these vertical systems intersect with multitude of horizontal territorial systems – from provinces to townships. So at every intersection, several organizations are involved for a single issue. The possible ways to make decision and coordinate behavior include command by hierarchy, voting, and bargaining. As no single organization is superior over another and voting has been avoided, the system falls back on bargain where decisions are made by "rule of consensus" or mutual accommodation (Lieberthal and Lampton 1992; Shirk 1993, p.116; Lampton 2014, p.86). The positive side of the fragmentation of authority is it prevents overconcentration of power; and the negative side is it makes achieving consensus very difficult (Shirk 1993, p.127). Each vertical organization is supposed to represent its constituents and pushes for policies in their own interests, but there are inadequate horizontal mechanisms of coordination, so they often find themselves at loggerheads or gridlock. If some organizations refuse to compromise and agreement cannot be reached, the issue is either dropped or is referred to a higher level for resolution (Shirk 1993, p.116). This is decision rule is also call "management by exception" by management specialists (Lawler 1976). Usually, the excessive amounts of issues to be solved are over the higher level authorities' capacity limit. Therefore, in Chinese public organs, too many problems remain unsolved for a long time simply because of the objection from a minority (Chen 1987, Lampton 1992, p.73). Although the "Fragmented Authoritarianism" framework changes the simply image of the top-down policy process, the problem is it's mainly a descriptive narration of the situation of domestic politics in China. It is still far from a theoretically and empirically grounded analytical framework, based on which we can make predictions.

There do exist plenty of theoretical studies of China's governance, but most of these

[4]The typical volums on this include: Policy Implementation in Post-Mao China, ed. David Lamptom 1987; Policy Making in China: Leaders, Structures, and Processes, Lieberthal and Oksenberg, 1988; Bureaucracy, Politics, and Decision Making in Post-Mao China, eds., Liebertal and Lampton, 1992.

studies focus on the vertical relation between the upper level governments and the lower level governments. Based on the theories of multidivisional structure of large corporations, the governance structure of China is modeled as a multiregional governance form (M-form) (e.g. Qian and Xu 1993; Eric Maskin, Qian, and Xu 2000; and Qian, Roland, and Xu 2006, 2007). In the M-form hierarchy, every region is controlled by the central government politically, whereas each region not only enjoys a certain degree of autonomy but also is self-contained in its functions (Xu 2011). Such structural arrangements generate high-powered incentives associated with regional competition which contributes to the improved quality of public fiscal policy in China, as China is often seen as a major example of market-preserving fiscal federalism (Gabriella Montinola, Qian, and Weingast 1995; Qian and Weingast 1997; Hehui Jin, Qian, and Weingast 2005). But the moral hazard problem is still prevalent in the principle-agent relationship between the center and the local. Such problems may range from the soft budget constraint syndrome (Qian and Roland 1998), to the local governments acting in ways that corrode the capacities of the central state (Hongbin Cai and Daniel Treisman 2004, 2005). The most effective tool the central leaders can use to control the lower level officials is the power of personnel appointment. For vertical control in Chinese bureaucracy system, a performance evaluation system is applied to coordinate governmental departments. Appointment, promotion and demotion of lower level bureaucrats are decided by whether they have fulfilled the upper level government's requirements for various policy targets. It is also observed in the literature that under the performance evaluation system, local officials in China are likely to be promoted on the basis of growth rate of GDP and fiscal revenue (Landry 2008, Li and Zhou 2005, Shih, et al. 2012). Different from the above studies which focus on the vertical central-local relation in China, our research addresses the horizontal coordination (logrolling) among the units within Chinese government, which has been under-researched in the existing theoretical literature.

However, logrolling agreement may be also problematic if too many groups are involved and if there is no external institution to guarantee the enforcement of the agreement, because the willingness to trade votes on one issue depends on the trust of others to pay back on the next issue (Shirk 1993, p.127). The earlier literature on logrolling doesn't consider the issue of enforcement, neither the time dimension in the implementation; all trades occur within one legislative session. Weingast and Marshal (1998) bring inter-temporal enforcement into the picture and, demonstrate that the committee system in congress also serves as enforcement. We think, in China, the inter-temporal dimension is also relevant in enforcing the logrolling deals. After studying policy coordination in China during the 1980s, Halpern (1992) points out the long-term planning efforts altered the policy process in ways that effectively placed the participants in a repeated-game situation, incentives were thereby created to cooperate now in exchange for future cooperation. Moreover, some of that future cooperation could be institutionalized in the present when policy documents were formulated that planned ahead many years (Halpern, 1992). This is similar to transaction in an economic market, in which each ministry may use policy documents as a kind of contract to establish and protect their "political property rights" away from the discretion of bureaucrats (Moe 1990). For example, in the guideline for social assistance released by Chinese government in May 2014, the role and responsibility of different government department are stated explicitly in this guideline for the forth-

coming expansion of social assistance programs. However, the enforcement of logrolling agreement is not the core issue of the current paper, the discussion here tries to justify our assumption in the following theoretical model that all the players commit to the logrolling deal.

3. The Theoretical Model

Consider a society with distinct but homogeneous interest groups $I \in \{\alpha, \beta\}$, each interest group can be seen as a single player α and β. Group α has preferences,

$$U^{\alpha}(h, x) = h + C(x), \tag{1}$$

and group β has preferences,

$$U^{\beta}(h, y) = h + C(y), \tag{2}$$

where h is general public good. The increasing and concave function $C(.)$, with $C(0) = 0$, is defined over the spending on group specific good $x \in [0, 1]$ or $y \in [0, 1]$. As a special example, suppose $C(\cdot) = \sqrt{(\cdot)}$.

$$x + y + h \leq 1.$$

3.1. Decision Made By Benevolent Social Planner

First, we derive the efficient benchmark. Suppose the allocation decision is made by a utilitarian social planner whose goal is to maximize social welfare W, then it maximizes the utilitarian social welfare function subject to the resource constraint $x + y + h \leq 1$:

$$max_{(h,x,y)} U^{\alpha}(h, x) + U^{\beta}(h, y) = max_{(h,x,y)} \left(2h + \sqrt{x} + \sqrt{y}\right) \tag{3}$$

$$s.t. \;\; x + y + h \leq 1; x \geq 0; y \geq 0; h \geq 0. \tag{4}$$

Since both utility functions are strictly increasing in x, y and h, the public budget constraint is binding which implies

$$h = 1 - x - y. \tag{5}$$

Hence, the social welfare problem is

$$max_{(x,y)} \left(2 - 2x - 2y + \sqrt{x} + \sqrt{y}\right) \tag{6}$$

$$s.t. \; x + y \leq 1, x \geq 0; y \geq 0. \tag{7}$$

The objective function is concave and the constraint is linear, therefore the Kuhn-Tucker conditions are both necessary and sufficient.[5]

The solution of the problem is,

$$(h^*, x^*, y^*) = \left(\frac{7}{8}, \frac{1}{16}, \frac{1}{16}\right).$$

This allocation is efficient as it maximizes the utilitarian social welfare.

[5]The full set of the Kuhn-Tucker conditions is given in the appendix.

3.2. Decision Made By One Interest Group

Then suppose the policy decision is made by either one of the interest groups. If group α has the decision making power, it will simply maximize its group welfare,

$$max_{(h,x,y)} U^{\alpha} = h + \sqrt{x} \tag{8}$$

$$s.t. \;\; h + x + y \leq 1; \; h \geq 0, \; y \geq 0, \; x \geq 0. \tag{9}$$

As α's objective function does not depend on y, then

$$y^{\alpha} = 0.$$

Moreover, since α's utility functions is strictly increasing in x and h, the public budget constraint is binding which implies

$$h = 1 - x. \tag{10}$$

Hence, the
maximization problem is

$$max_x \left(1 - x + \sqrt{x}\right) \quad s.t. \; x \geq 0, \; x \leq 1. \tag{11}$$

The objective function is concave and the constraint is linear, therefore the Kuhn-Tucker conditions are both necessary and sufficient[6]:

The solution of the problem is, if α has full power to choose its preferred allocation, then it would implement

$$(h^{\alpha}, x^{\alpha}, y^{\alpha}) = \left(\frac{3}{4}, \frac{1}{4}, 0\right).$$

Symmetrically, if β has the decision making power, it will choose

$$\left(h^{\beta}, x^{\beta}, y^{\beta}\right) = \left(\frac{3}{4}, 0, \frac{1}{4}\right).$$

Compare the result with the policy made by the benevolent social planner, we can see

$$h^{\alpha/\beta} < h^*, x^{\alpha} > x^*, y^{\alpha} < y^*, x^{\beta} < x^*, y^{\beta} > y^*.$$

It means there is suboptimal amount of public good because of the excessive spending on its preferred issue by the group who has the decision making power.

3.3. Decision Made By Bargaining

Next we focus on the bargaining process, adapting the simplest legislative bargaining process discussed in the seminal work by Baron and Ferejohn (1989), Persson (1998), Persson and Tabellini (2000). The bargaining follows the following sequence of events:

1. one of the interest groups, say α (we can also assume β, but the analysis is totally symmetric), is chosen to be the agenda setter;

[6]The full set of the Kuhn-Tucker conditions is given in the appendix.

2. the agenda setter makes a policy proposal (h, x, y);

3. the other group β chooses to accept the proposal or not. If it accepts the proposal made by the agenda setter then the proposal is implemented; if not, a default outcome $(h, x, y) = (1, 0, 0)$ gets implemented, which makes it to receive a reserve utility of $U^S = 1$.

From a formal point of view, this is equivalent to an ultimatum game, with usual standard properties.We can work out this simple game backward. The other group will reject the proposal made by the agenda setter if it not getting at least as high a payoff from the policy proposal as from the default policy (we assume throughout that the other group will accept a proposal when indifferent between it and the alternative).

Hence, the other group will accept the proposal if and only if

$$U^\beta - U^S = 1 - x - y + \sqrt{y} - 1 \geq 0 \iff x \leq \sqrt{y} - y. \tag{12}$$

Knowing this, the agenda setter α will maximize its own utility (1) subject to the "incentive constraint" (12). Thus, the agenda setter α chooses to make a policy proposal according to the following programming.

$$max_{(x,y)} U^\alpha = 1 - x - y + \sqrt{x} \tag{13}$$

$$s.t. \;\; x \leq \sqrt{y} - y; x + y \leq 1; x \geq 0; y \geq 0. \tag{14}$$

The objective function is concave and the constraints are convex, therefore the Kuhn-Tucker conditions are both necessary and sufficient[7].

The solution of the problem is, if α is first-mover, then she would implement

$$\left(h^{\alpha B}, x^{\alpha B}, y^{\alpha B}\right) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{1}{8}, \frac{3}{8} - \sqrt{\frac{1}{8}}\right).$$

Symmetrically, in a bargaining setting if β is the first-mover, then he will choose

$$\left(h^{\beta B}, x^{\beta B}, y^{\beta B}\right) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{3}{8} - \sqrt{\frac{1}{8}}, \frac{1}{8}\right).$$

Therefore, when decision is made by bargaining, the agenda setter has the first-mover advantage.

Compare the result with the policy made by the benevolent social planner and the interest group itself, we can see

$$h^{\alpha/\beta} < h^{\alpha/\beta B} < h^*, x^\alpha > x^{\alpha B} > x^*, y^\alpha < y^{\alpha B} < y^*, x^\beta < x^{\beta B} < x^*, y^\beta > y^{\beta B} > y^*.$$

It implies a suboptimal allocation of x, y and h, because of an excessive spending on its preferred issue by the group having the agenda setting power. However, the distortion and overreaching are reduced with respect to the case when the interest group has the monopoly power on policy making.

[7]The full set of the Kuhn-Tucker conditions is given in the appendix.

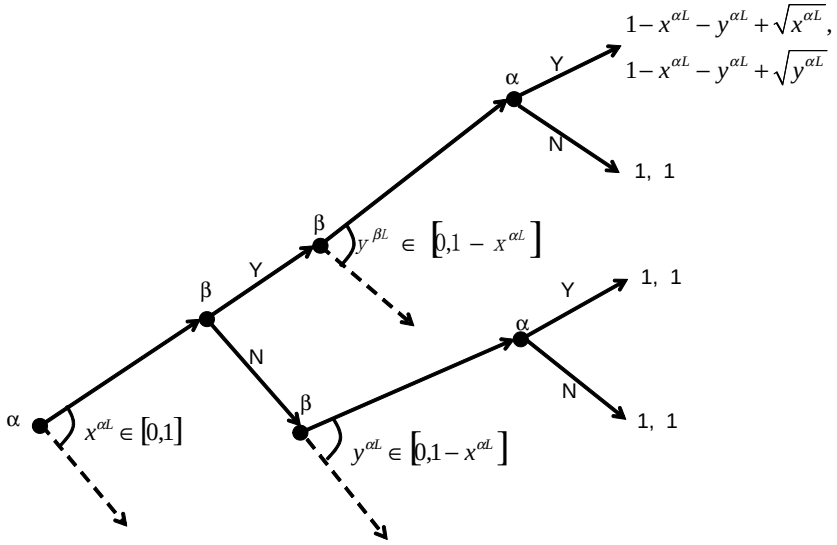

Figure 5. The game of logrolling

3.4. Decision Made By Logrolling

Next, we consider the simplest logrolling process:

1. α[8] proposes a motion $x^{\alpha L} \in [0,1]$;
2. β chooses whether to support or not $\alpha's$ proposal, i.e. $c^{\beta} \in \{Y, N\}$;
3. β proposes a motion $y^{\beta L} \in [0, 1 - x^{\alpha L}]$;
4. α chooses whether to support or not $\beta's$ proposal, i.e. $c^{\alpha} \in \{Y, N\}$;
5. If the two players supported each other's proposal, i.e. $c^{\beta} = c^{\alpha} = Y$, we say a logroll is forged, and the proposals $\left(1 - x^{\alpha L} - y^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right)$ in the logroll are implemented;
6. otherwise, if any player rejected the other, we say the logrolling failed. Then a default outcome $(h, x, y) = (1, 0, 0)$ gets implemented, hence both players will get the reserve utility

$$U^S = 1. \tag{15}$$

The following picture represents the game tree:

We can work out this game from backward.

[8]We can let β moves first, but the analysis is totally symmetric.

3.4.1. STEP 4:

In the last decision step, α chooses between to accept the policy proposed by β and the default policy. We should distinguish two different set of subgames, whether in step 2 β rejected or accepted α's proposal. If $c^\beta = N$, then α's choice is irrelevant as both players will get the default outcome. If $c^\beta = Y$, then $c^\alpha = Y$ if and only if

$$U^\alpha\left(Y|h^3 = (x, Y, y)\right) \geq U^S \iff 1 - x - y + \sqrt{x} \geq 1 \iff y \leq \sqrt{x} - x \tag{16}$$

The condition defined by inequality (16) is α's "incentive logrolling (IL) constraint".

Hence, we get the following sequential best reply for α:

$$SBR^\alpha(h^3) = \begin{cases} \in \{S, N\}, if\ h^3 = (x, N, y) \\ N,\ if\ h^3 = (x, Y, y)\ s.t.\ y \geq \sqrt{x} - x \\ Y,\ if\ h^3 = (x, Y, y)\ s.t.\ y \leq \sqrt{x} - x \end{cases}. \tag{17}$$

3.4.2. STEP 3:

In step 3, β will choose $y^{\beta L}$ to maximize his utility subject to $SBR^\alpha = Y$. Again, we should distinguish two different set of subgames, i.e. whether in step 2 β rejected or accepted α's proposal. If $c^\beta = N$, then β's choice is irrelevant as both players will get the default outcome. Hence

1. if $h^2 = (x, N)$, then $SBR^\beta(h^2) \in [0, 1]$;
2. if $h^2 = (x, Y)$, then $SBR^\beta(h^2) \in argmax_y U^\beta\left(y|h^2 = (x, Y), SBR^\alpha(h^3)\right)$.

If $y^{\beta L} \geq \sqrt{x} - x$, then α will not choose to logroll with β, and β can only get the reserve utility, i.e. $U^\beta\left(y^{\beta L}|h^2 = (x, Y), SBR^\alpha(h^3)\right) = U^S = 1$.

Otherwise, β will solve following maximization problem:

$$max_y\left(1 - x - y + \sqrt{y}\right) \tag{18}$$

$$s.t.\ y \leq \sqrt{x} - x \qquad \text{(IL constraint)}$$

$$y \leq 1 - x \qquad \text{(Resource constraint)}$$

$$y \geq 0 \tag{19}$$

Note that in the above problem IL constraint is more restrictive than the resource constraint, hence we can omit it. The objective function $1 - x - y + \sqrt{y}$ is great than 1 if and only if $x \leq \frac{1}{4}$. Moreover, $\beta's$ objective function has a maximum when $y = \frac{1}{4}$, which is great or equal than$\sqrt{x} - x$ for any x. Therefore, the solution of the sequential best response of β is

$$SBR^\beta(h^2) = \begin{cases} \sqrt{x} - x & if\ h^2 = (x, Y)\ \&\ x \in \left[0, \frac{1}{4}\right] \\ \in [\sqrt{x} - x, 1] & if\ h^2 = (x, Y)\ \&\ x \in \left[\frac{1}{4}, 1\right] \\ \in [0, 1] & if\ \ h^2 = (x, N). \end{cases}$$

3.4.3. STEP 2

In step 2, β will choose whether to support α or not. In particular, $c^\beta = Y$ if and only if

$$U^\beta(Y|x, SBR^\beta(h^2), SBR^\alpha(h^3)) \geq U^S \tag{20}$$

which implies $x \leq \sqrt{SBR^\beta(h^2)} - SBR^\beta(h^2)$.

Hence, we need to distinguish two cases, $x \in \left[0, \frac{1}{4}\right]$ and $x \in \left[\frac{1}{4}, 1\right]$.

Case $x \in \left[0, \frac{1}{4}\right]$

In this case β will choose $c^\beta = Y$ if and only if

$$x \leq \sqrt{SBR^\beta(h^2)} - SBR^\beta(h^2) \tag{21}$$

which implies

$$x \leq \sqrt{\sqrt{x} - x} - \left(\sqrt{x} - x\right). \tag{22}$$

(22) is always satisfied for any $x \in \left[0, \frac{1}{4}\right]$.

Case $x \in \left[\frac{1}{4}, 1\right]$

In this case $U^\beta(Y|x, SBR^\beta(h^2), SBR^\alpha(h^3)) = U^S$, hence $c^\beta \in \{Y, N\}$.

From both cases we get the following sequential best reply for β:

$$SBR^\beta(x) = \begin{cases} Y & if\ x \in \left[0, \frac{1}{4}\right] \\ \in \{Y, N\} & if\ x \in \left[\frac{1}{4}, 1\right]. \end{cases} \tag{23}$$

3.4.4. STEP 1

Moving backward, in step 1, α will choose $x \in [0, 1]$ to maximize $U^\alpha(x|SBR^\beta(x), SBR^\beta(h^2), SBR^\alpha(h^3))$ which is

$$U^\alpha(x|SBR^\beta(x), SBR^\beta(h^2), SBR^\alpha(h^3)) = \begin{cases} 1 - x - \sqrt{x} + x + \sqrt{x} = 1 & if\ x \in \left[0, \frac{1}{4}\right] \\ 1 & if\ x \in \left[\frac{1}{4}, 1\right]. \end{cases}$$

Thus $U^\alpha(x|SBR^\alpha(x), SBR^\beta(h^2), SBR^\alpha(h^3))$ is constant and equal to 1,hence $SBR^\alpha \in [0, 1]$. To find a solution, we assume that there is a lexicographic preference for forging an agreement, hence $SBR^\alpha = \frac{1}{4}$.

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

3.4.5. Comments:

We can put policy outcomes from difference decision-making rules together in order to make comparision.

- The efficient allocation is $(h^*, x^*, y^*) = \left(\frac{7}{8}, \frac{1}{16}, \frac{1}{16}\right)$;
- When decision making is by only one group (when one group dominates the other), the allocation is $(h^\alpha, x^\alpha, y^\alpha) = \left(\frac{3}{4}, \frac{1}{4}, 0\right)$ or $\left(h^\beta, x^\beta, y^\beta\right) = \left(\frac{3}{4}, 0, \frac{1}{4}\right)$;

- When decision making is by bargaining, $\left(h^{\alpha B}, x^{\alpha B}, y^{\alpha B}\right) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{1}{8}, \frac{3}{8} - \sqrt{\frac{1}{8}}\right)$ or $\left(h^{\beta B}, x^{\beta B}, y^{\beta B}\right) = \left(\frac{1}{2} + \sqrt{\frac{1}{8}}, \frac{3}{8} - \sqrt{\frac{1}{8}}, \frac{1}{8}\right)$. There is first-mover advantage;
- When decision making is by logrolling, $\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$. There is no first-mover advantage.

Compare the allocation of different issues under logrolling with the previous allocations, we can see

$$\begin{aligned} h^{\alpha L} &= h^{\beta L} < h^{\alpha/\beta} < h^{\alpha/\beta B} < h^*, \\ x^{\alpha} &= x^{\alpha L} > x^{\alpha B} > x^*, \\ y^{\alpha} &< y^{\alpha B} < y^* < y^{\alpha L}, \\ x^{\beta} &< x^{\beta B} < x^* < x^{\alpha L}, \\ y^{\beta} &= y^{\alpha L} > y^{\beta B} > y^*. \end{aligned}$$

There are several interest points worth to mention. First, the allocation of x, y and h is suboptimal, but the distortion caused by logrolling is the most serious, as public good provision is the smallest under logrolling. The direction of policy distortion is different from other cases. Under logrolling, there is excessive spending on both x and y, the two issues through which the interest groups trading favor with each other. It means the problem of policy overreaching is more serious under logrolling. Second, the first-mover advantage we observed in the case of bargaining no longer exists in the case of logrolling. The first-mover advantage in the exisiting legislative bargaining literature is treated as the agenda setting power (Persson, 1998). But when decision making is by logrolling, there is no first-mover advantage. This finding helps to understand the fundamental difference between logrolling and bargain. The game under bargaining is a strictly competitive game, in which one player's gain is at the cost of the other player's welfare. Therefore, the player who is chosen to move first will exploit all the advantages. However, the game under logrolling is not a strictly competitive game, where the sum of the players' utility changes with their strategy. Coordinating well, they can increase their joint benefit at the cost of society's welfare, as there is under-provision of public goods. This is the element cooperation in the logrolling which does not exist in bargaining. Therefore, under logrolling, the each player's welfare is internalized, and hence, it doesn't matter which player moves first.

3.5. Robustness check

In the above calculations, we assume when logrolling fails a default outcome $(h, x, y) = (1, 0, 0)$ will be implemented. In order to check whether the outcome of logrolling depends on the default policy, we will generalize the default outcome. We can assume when logrolling failed both players will get a reserve utility $U^D = \theta \in [0, 1]$.

The "incentive logrolling (IL) constraint" for α then becomes

$$U^{\alpha}\left(Y|h^3 = (x, Y, y)\right) \geq U^D \tag{24}$$

which implies

$$1 - x - y + \sqrt{x} \geq \theta \tag{25}$$

i.e. the other group will reject the logrolling proposal if it not getting at least as high a payoff from the default policy. In equilibrium the player's payoff from logrolling is ever greater or equal than the default policy, therefore the players always have incentive to forge a logrolling deal and the value of the default policy is irrelavant from the outcome path[9].

Proposition 1 *In the logrolling game, assuming that both agents' utility when the logrolling fails is*

$$U^D = \theta \in [0, 1], \tag{26}$$

the outcome path is independent from θ *and is*

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

4. The Empirical Evidence

In this section, we illustrate empirical evidences of logrolling among the functional vertical organizations in China, which results in over-reaching and inefficient policy outcomes. The logrolling is taking place between the Ministry of Civil Affairs (MCA) and the Ministry of Health (MOH) in the policy arena concerning social assistance.

In our example, the two negotiators, MCA and MOH, must choose government policies with respect to three different issues, i.e. Dibao, Rural health insurance, and Mental Health Care. For logrolling to be possible, the negotiators must have different priorities on different issues. The two ministries' preference on different kind of goods is shown in Figure 6. MCA's top priority is Dibao, and MOH's top priority is rural health insurance. If each ministry sticks to its own preferences in these issues, they cannot reach consensus. According to "rule of delegation by consensus" described in the citation at the beginning of paper, these issues will be "tabled" or are referred to a higher level for resolution. Then the authorities step in to make the decision, so that the default policy is implemented.

If the two ministries trade favor, they could form a logrolling coalition and secure the policy issue of their high priority. The MOH supports the increase of the benefits of Dibao and bears extra cost, but it anticipates MCA will support its core interest in rural health insurance. In exchange, the MCA, will pay for the insurance premium for poor households in rural areas in order to support MOH's core interest in expanding coverage of health insurance. Mental health care, which is not the priority for neither of the two ministries, will be sacrificed. Figire 7 shows how ministries can exchange interests via various policies.

4.1. Inefficiency in the high benefit associated with "Dibao"

Because of logrolling, there is overreaching in the benefit associated with "Dibao" such that it even crowds out social insurance. Here we show the evidence of crowding out between Dibao and social insurance. We use two city-level datasets with data of social assistance programs over 280 cities in China between year 2003 and 2009. We find

[9]The detailed prove is given in the appendix.

Figure 6. Preference of the ministries

	High Priority	**Low Priority**	**Public Good**
MCA	**Dibao** *Providing as much financial and other assistances for households tagged as "Dibao" households as possible.*	**Rural Health Insurance*** *Regarding to poor rural households, MCA has incentive to only insure high risk people with rural health insurance but not the low risk people.*	**Mental Health Care** *Treating patients in MCA managed hospitals*
MOH	**Rural health insurance** *Expanding coverage of health insurance. Increasing the number of enrollees.*	**Dibao** *MOH sets the standard for receiving medical assistance. Whether a recipient under Dibao or not is not very important for MOH*	**Mental Health Care** *Treating patients in MOH managed hospitals*

*Note：Different from social health insurance in developed counties, rural health insurance in China is a voluntary health insurance.

Figure 7. Logrolling via various policies

	Dibao	**Rural health insurance**	**Mental Health Care**
MCA		MCA uses the assistance fund to pay for the insurance premium for poor households in rural areas to register with rural health insurance plan.	Insufficient input for infrastructure building
MOH	Dibao households automatically are included as recipients for medical assistance.		Insufficient input for infrastructure building

that increasing social assistance expenditure are likely to reduce the number of registered unemployed, which implies some urban residents claiming social assistance are either leaving the labor market or had evaded from contribution for the unemployment insurance. Many urban residents do not enroll with unemployment insurance because the benefits from Dibao and other social assistance programs are higher than the benefit from unemployment insurance. Our finding suggests that the crowding out effect is a result of logrolling.

4.1.1. Hypotheses

While we do not have the data for the number of enrollees of unemployment insurance in each city, we use the number of registered unemployed as a proxy for the number of enrollees of unemployment insurance in the informal sector. While the number of enrollees has increased in the formal sector, workers in the informal sector may choose to enroll with unemployment insurance or not. Since only people who have registered as unemployed can claim the unemployment insurance, we can infer the number of unregistered unemployed in the following year is positively correlated with the number of enrollees of unemployment insurance in the current year.

We have following two hypotheses regarding to whether the regional equity has been addressed in the urban social assistance programs:

Hypothesis 1: increasing government expenditure on "Di Bao" is associated with a smaller number of registered unemployed in a city.

Hypothesis 2: increasing government expenditure on social assistance programs in general is associated with a smaller number of registered unemployed in a city.

4.1.2. Data

We have two data sources. The first data set is collected from China City Statistical Yearbook (NBS: various years). This dataset includes observations of 282 cities (prefecture level) covering 26 out of 27 provinces between 2003 and 2006 (i.e. Tibet is not included). These four years (i.e. from 2003 to 2006) are only years for which the data for spending on social assistance is available. The second dataset is data reported by MCA about Dibao data at the prefecture level between 2007 and 2009, which was collected from the website of MCA.

Both of our datasets include data for 282 prefecture cities out of 332 prefecture cities in total in China. City statistical yearbook only reports data in 282 cities. A prefecture city usually has both urban (i.e. city district) and rural areas (i.e. county). Since we are interested in urban social assistance programs, most variables in our dataset are defined in the scale of city district of these prefecture cities only.

4.1.3. Methodology

Research on crowding-out effect uses individual level data. We do not have the luxury to access such data in China. However, we use city level data to treat a city rather than individuals as the basic research unit. City level data may reveal people's response to social programs since city level government is the level of government managing the social programs and policies regarding to the benefit and eligibility for social programs varies with city.

However, omitted variable bias is a concern for this kind of research. We have apply

(city) fixed effect model to address this issue. Simultaneity is another concern. We use predetermined values for control and independent variables to address this issue.

Nevertheless, we acknowledge that city level data cannot tell us what individual characteristics are more likely to lead to opt out from unemployment insurance. Also, some concerns about the measurement of unemployment. For a long time, China only reports the registered unemployed and actual unemployed number is absent. However, for this paper, registered unemployed number is a sufficient indicator for the crowding-effect for the unemployment insurance. Only registered unemployed can be eligible to claim unemployment insurance.

It may be argued that the number of unemployed may be reduced via other avenues such as exogenous shocks of government policies or new policy initiatives which have impact on urban unemployment. In this case, we use local education expenditure as a regressor in the placebo test. If the number of unemployed is not correlated with education expenditure, it confirms the existence of crowding-out effect.

4.1.4. Basic Model

We estimate the following model:

$$Unemloyed_{i,t} = \beta poverty\_assistance_{i,t-1} + \delta X_{i,t} + \mu_i + \omega_t + e_{i,t} \tag{27}$$

where $Unemployed$ is the number of registered unemployed per 1000 people in city i during year t. β and δ are parameters for the corresponding variable(s) in the model. $X_{i,t}$ corresponds to covariates including average income, fiscal expenditure per capita, size of service economy, local gross product as well as unemployed in the previous year. μ_i denotes province-specific effects, whereas ω_t corresponds to year dummy variables, with $e_{i,t}$ defined as the error term. We use standard panel data analysis to estimate above model.

If there is a crowing out effect between social assistance programs and unemployment insurance, urban residents may have not enough incentive to enroll with unemployment insurance. In consequence, urban unemployed have not enough incentive to register as unemployed.

4.1.5. Variables

The dependent variable is Unemployment: unemployment refers to the number of registered unemployed in the city district of a city (In 1,000 people). Many of the people claiming Dibao are registered unemployed. Figure 2 shows the number of unemployed who are under Dibao program, which amounts to about 20% of beneficiaries who are under social assistance programs. Control and independent variables are listed as following:

Poverty_assistance denotes the city level spending on social assistance programs (in Million RMB). The amount of poverty assistance expenditure includes all expenditure including Dibao and other complementary social assistance (Figure 8).

Dibao_expenditure denotes the city level spending on Dibao programs (in Million RMB).

Variable "population" denotes the number of residents in the city district of a city (Million population).

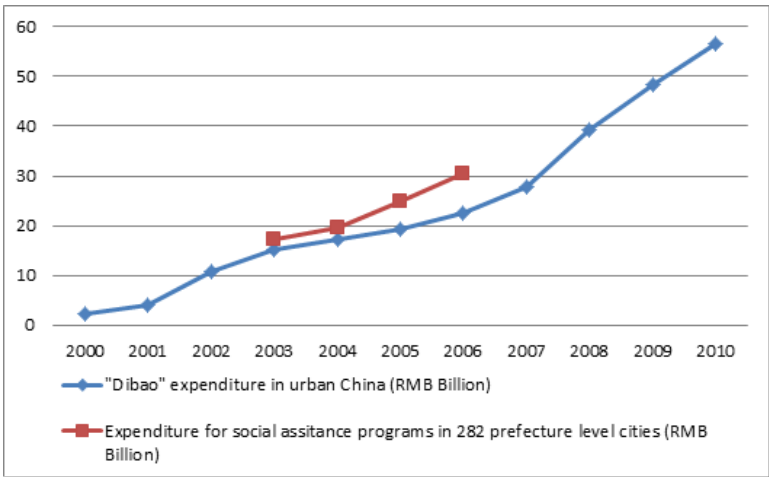

Figure 8. "Dibao" and social assistance expenditure in urban China (Billion RMB)

Fiscal expenditure of a prefecture city (in Billion RMB). Fiscal expenditure measures the scale of local fiscal policies and in this case are supposed to be negatively correlated with the number of registered unemployed .

Average income: this variable refers to the average annual wage level in the city district of a city. The average wage is calculated on the basis of wage expenditure in state owned, privately owned as well as foreign owned enterprises. This variable measures local conditions of economic development.

Gross product: this variable denotes the gross regional product in a prefecture city (Billion RMB). This variable also measures local conditions of economic development but also measures the size of economy.

Service share: the share of local service sector in gross regional product. The share of service sector is not directly linked to the social assistance expenditure. However, service sector is believed to be very labor intensive and the larger the service sector is, the more people are hired. Consequently, the number of registered unemployed is supposed to be negatively correlated with the size of service sector.

Workers in private and informal sector: This variable refers to the number of people working in private and informal sector in the city district of a city. This variable is relevant since we expect a larger private and informal sector may imply more people who are not willing to register as urban unemployed. Private and informal sector are relevant for urban unemployment in China. For example, 8% of poor's incomes are earned from private and informal sector and only 2.7% of incomes of other groups are earned from private and informal sector (Riskin and Gao 2010).

City dummy, year dummy as well as dummy variables for western and central regions. Western and central regions are defined following the definition in the China Statistical Yearbook .

Figure 9.

Variable	Obs	Mean	Std. Dev.	Min	Max
Expenditure on social assitance (Million RMB)	1118	81.56	106.34	4.29	978.20
Fiscal expenditure (Billion RMB)	1118	2.7	4.65	0.14	57.1
Average income (Thousands RMB)	1117	15.97	5.02	4.93	37.81
Gross Regional product (Billion RMB)	1116	29.85	51.37	0.98	581.36
Population (Million)	1118	1.11	0.99	0.14	8.01
Unemployment (Thousands)	1118	12.74	16.62	0.22	157.23
Share of service sector (%)	1115	40.84	10.32	9.87	80.89
Workers in Private/informal Sector (thousands)	1109	125.11	208.61	1.10	2673.98
Domestically owned enterprises (units)	1115	265.68	480.56	3	4491
Education expenditure (RMB billion)	1108	0.35	0.54	0.001	5.76
***Dibao* expenditure (RMB million)**	832	105.44	76.50	3.57	481.64

4.1.6. Descriptive statistics

We exclude some invalid observations from our sample and the descriptive statistics are shown in figure 9.

The expenditure for social assistance has increased by over double digit annual growth from less than RMB 20 billion to over RMB 30 billion between 2003 and 2006 in the 282 cities in our dataset. However, regional variances of the government expenditure on social assistance as well as other covariates are huge. The highest level of government expenditure is as high as RMB 1 billion while the lowest is only RMB 4 million. In the richest city, the annual average income is RMB 37 thousands while in the annual average income is RMB 5 thousands the poorest city. The fiscal revenue of richest city reached RMB 50 billion and fiscal revenue in the poorest city had only RMB 0.1 billion. Figure 1 shows that the expenditure for social assistance programs in these 282 prefecture cities has increased from less than RMB 20 billion to more than RMB 30 billion in four years.

4.1.7. Results

The estimation result for equation (1) is shown in Figure 10:

Column (1) and (2) report results of random effect model and city fixed effect model respectively. In Column (3) and (4), all variables are measured in per capita basis. The result of Hausman test suggests that fixed effect models in (2) and (4) are consistent. In

	(1) Unemployment (RE)	(2) Unemployment (FE)	(3) Unemployment Per 1000 people (RE)	(4) Unemployment Per 1000 people (FE)
fiscal	-0.0177	-0.552	-0.116	-0.352
expenditure	(0.171)	(0.378)	(0.267)	(0.645)
Social assistance expenditure (lagged)	0.0217***	-0.0204**	0.0236***	-0.0184*
	(0.00506)	(0.00927)	(0.00600)	(0.0101)
population	2.000***	1.240		
	(0.389)	(1.166)		
Average	0.0586	-0.0601	-0.0365	-0.0472
income	(0.0569)	(0.127)	(0.0656)	(0.149)
unemployment	0.650***	0.151***	0.523***	0.134***
(lagged)	(0.0196)	(0.0334)	(0.0221)	(0.0327)
Private	0.00254	0.00635	0.00267	-0.00246
workers	(0.00202)	(0.00424)	(0.00356)	(0.00665)
Service share	-0.0321	0.0209	0.0288	0.0548
	(0.0347)	(0.0889)	(0.0568)	(0.162)
Gross product	0.00523	0.0278	0.0174	0.0529
	(0.0196)	(0.0532)	(0.0252)	(0.0868)
2006	-2.541***	-0.193	-2.364***	-1.270
	(0.659)	(0.987)	(0.751)	(1.286)
2005	-1.335**	-0.261	-1.327*	-1.022
	(0.596)	(0.707)	(0.679)	(0.899)
2004	-2.093***	-1.292**	-2.262***	-1.803***
	(0.572)	(0.572)	(0.651)	(0.686)
western	0.232		-0.142	
	(0.547)		(0.625)	
central	1.138**		0.972	
	(0.530)		(0.602)	
constant	0.279	11.91***	4.630***	12.01***
	(1.070)	(2.365)	(1.178)	(2.504)
N	1114	1114	1114	1114
R^2	0.806	0.514	0.428	0.185

Standard errors in parentheses, * p<.1, ** p<0.05, *** p<0.01 For Colume (3) and (4), all regressors

Figure 10. Results

model (2) and (4), the expenditure on social assistance in general is negatively and statistically significant. From column (2), one more million RMB spent on social assistance are associated with 20 less registered unemployed. The magnitude of the impact of the social assistance over unemployment is similar in column (4) when variables are weighted by the number of population. This implies the existence of crowding out effect that increasing expenditures of social assistance are likely to decreased registered unemployed.

However, random effect model in column (1) show that the effect of time invariant variables: central region dummy is statistically significant. Fiscal expenditure is not statistically significant in all models in Table 10. Average income and gross regional product are not statistically significant in all models. Population is statistically significant in model (1) and previous year's unemployment is important in all models.

Where is the crowding out effect coming from? The crowding out effect can be a result of expanding of Dibao or the expanding of Dibao and other complementary social assistance programs. To verify whether it is the Dibao that crowded out the unemployment insurance, we replace the expenditure on social assistance with the expenditure on Dibao as a regressor. Since data for Dibao expenditure covers only between 2007 and 2009, we have 561 observations in total.

The result is shown in table 11. Column (1) and (2) in table 11 report results of random effect model and city fixed effect model respectively. In Column (3) and (4), all variables are measured in per capita basis. The result of Hausman test suggests that fixed effect models in (2) and (4) are consistent. The expenditure on Dibao is not statistically significant in all models, which implies that the expansion of Dibao is not correlated with registered unemployed.

In short, H1 is not supported while H2 is supported from our data analysis.

4.1.8. Robustness check

The decreased registered unemployed may be caused by reasons other than crowding out effect. It may be a result of increasing government social expenditure such as education or health, which may have impact on people's welfare and participation of labor market.

To exclude the possibility that the changes of the number of registered unemployed are caused by exogenous shocks or other reasons, we conduct another placebo test by replacing the social assistance expenditure with education expenditure. Education expenditure is supposed to not directly correlate with registered employed after controlling local fiscal expenditure.

Column (1) and (2) in table 12 report results of random effect model and city fixed effect model respectively. In Column (3) and (4), all variables are measured in per capita basis. The result of Hausman test suggests that fixed effect models in (2) and (4) are consistent.

Education expenditure is not statistically significant in column (2) and (4). It confirms the robustness of our earlier results that the expenditure on social assistance is crowding out the unemployment insurance.

4.2. Inefficiency in the enrollment of rural health insurance

Because of logrolling, there is overreaching in the enrollment of rural health insurance. Here we show the evidence of inefficiency in the enrollment of rural health insurance. We find that a wider insurance coverage driven by rural medical assistance fund is not

Figure 11.

	(1) Unemployment (RE)	(2) Unemployment (FE)	(3) Unemployment Per 1000 people (RE)	(4) Unemployment Per 1000 people (FE)
Dibao expenditure (lagged)	0.00456 (0.00565)	-0.0144 (0.0133)	0.00188 (0.00185)	0.00158 (0.00762)
Fiscal expenditure	0.260** (0.126)	-0.402 (0.314)	0.0204 (0.112)	-0.0876 (0.281)
population	1.169 (0.756)	0.242 (0.746)		
Average income	0.0811* (0.0426)	0.131** (0.0625)	0.0217 (0.0331)	0.0747 (0.0584)
Unemployment (lagged)	0.890*** (0.0324)	-0.162* (0.0856)	0.842*** (0.0261)	-0.123 (0.0870)
Private workers	0.00699*** (0.00182)	-0.00133 (0.00304)	0.00493** (0.00202)	0.000290 (0.00385)
population	0.324 (0.831)	6.668 (6.109)		
Service gdp	-0.198*** (0.0318)	0.0976 (0.126)	-0.0367 (0.0297)	0.0565 (0.133)
Gross product	0.0350** (0.0165)	0.0652 (0.0772)	0.00741 (0.0138)	0.0249 (0.0706)
2008	-0.511 (0.361)	-0.184 (0.505)	-0.235 (0.301)	-0.373 (0.404)
western	0.772 (0.682)		0.399 (0.459)	
central	0.864 (0.698)		0.730 (0.454)	
constant	-3.002** (1.202)	1.528 (7.357)	0.281 (0.849)	8.733*** (1.830)
N	561	561	561	561
adj. R^2	0.896	0.354	0.733	0.07

Standard errors in parentheses. * p<.1, ** p<0.05, *** p<0.01, For Colume (3) and (4), all regressors are weighted with population

Figure 12. Robustness check

	(1) Unemployment (RE)	(2) Unemployment (FE)	(3) Unemployment Per 1000 people (RE)	(4) Unemployment Per 1000 people (FE)
Fiscal expenditure	0.273	-0.814**	0.751**	-0.430
	(0.170)	(0.360)	(0.292)	(0.651)
Education expenditure (lagged)	-2.391*	-0.561	-7.968***	-0.536
	(1.292)	(1.572)	(2.410)	(3.751)
population	2.790***	1.460		
	(0.369)	(1.166)		
Average income	0.0801	-0.0953	-0.0200	-0.0626
	(0.0570)	(0.126)	(0.0656)	(0.150)
unemployment (lagged)	0.683***	0.158***	0.535***	0.135***
	(0.0179)	(0.0334)	(0.0216)	(0.0327)
Private workers	0.00289	0.00773*	0.00460	-0.00263
	(0.00206)	(0.00421)	(0.00357)	(0.00667)
Service gdp	0.00422	0.0208	0.0178	0.0557
	(0.0340)	(0.0893)	(0.0569)	(0.162)
gross product	-0.00593	0.0246	0.00993	0.0523
	(0.0198)	(0.0540)	(0.0250)	(0.0871)
2006	-2.031***	-0.301	-1.793**	-1.603
	(0.657)	(0.989)	(0.747)	(1.297)
2005	-1.011*	-0.298	-1.013	-1.204
	(0.598)	(0.709)	(0.677)	(0.905)
2004	-2.131***	-1.238**	-2.199***	-1.830***
	(0.576)	(0.573)	(0.652)	(0.690)
western	-0.0814		-0.837	
	(0.557)		(0.634)	
central	0.960*		0.516	
	(0.541)		(0.617)	
constant	-0.0531	11.53***	6.092***	11.50***
	(1.075)	(2.366)	(1.209)	(2.533)
N	1114	1114	1114	1114
adj. R^2	0.817	0.626	0.426	0.323

Standard errors in parentheses. * p<.1, ** p<0.05, *** p<0.01, For Colume (3) and (4), all regressors are weighted with population

associated with a higher utilization rate of health services. We use provincial level data to explore the relation between medical assistance fund and rural health insurance. The data source is from China Health Statistical Yearbook as well as China Civil Affairs Statistical Yearbook, Various years. The time span of our data is between 2007 and 2012.

Table 13: definition of variables

Variables	**Definition**
rural_medical assistance	Number of people receiving rural medical assistance per million populati
medical_assistance fund	The amount of medical assistance fund (RMB million per million popula
disposable_income	Disposable income RMB ,000 per year
inpatient	volume of inpatient service per million population
h_doctors	number of doctors per 1,000 population
rural_insurance	% of enrolees of rural insurance receiving medical assistance
urban_insurance	% of enrolees of urban insurance receiving medical assistance
age65	Share of population aged 65 or above
h_beds	No of hospital beds per 1000 people
bhi	% of enrolees under basic urban health insurance per 100 people
cms	% of enrolees under rural health insurance
urbmi	% of enrolees under urban residence health insurance

Table 14 Descriptive statistics

	count	mean	sd	min	max
rural_assistance	186	9368.713	13992.04	0	94472.11
fund_rural_assistance	185	702.298	655.941	5.070	3630.063
cms	184	55.164	18.519	0	83.918
disposable_income	186	17.7038	5.768	10.012	40.188
hospital_inpatients	186	753334.7	250605.9	285971.8	1542501
h_doctors	186	1.583	.508	.793	3.594
rural_insurance_assitance	184	3.0526	2.456	0	14.472
urban_insurance_assitance	124	1.299	1.214	.0102	5.808
age65	186	.090	.017	.048	.143
h_beds	186	3.595	.783	2.104	5.894
bhi	186	18.476	12.511	6.066	69.214
disposable_income	186	17.704	5.768	10.012	40.188
urbmi	124	15.132	10.652	2.864	89.970

First, we find that the number of enrollees, whose premium for rural health insurance paid by medical assistance fund, is not correlated with the number of people whose health expenditure is directly reimbursed by medical assistance fund.

This preliminary result (Table 15) suggests that two major usages of the medical assistance fund: paying for premium of rural health insurance and reimbursement of health expenditure, are not crowding out each other.

Table 15: Dependent variable: number of enrollees whose insurance premium is paid from rural medical assistance fund

	(1)
	rural_insurance_assistance
rural_assistance	-0.00000717
	(0.0000130)
fund_rural_assistance	0.00124***
	(0.000293)
cms	0.0204
	(0.0162)
disposable_income	0.0470
	(0.0334)
constant	0.312
	(1.089)
N	183
adj. R^2	0.174

Standard errors in parentheses

* p<.1, ** p<0.05, *** p<0.01

However, the following regression table 16 show that the number of people subsidized for paying premium of rural health insurance is not associated with the revenue, volume of inpatient services in hospitals as well as outpatients services. These regression results suggest that a wider insurance coverage driven by rural medical assistance fund is not associated with a higher utilization rate of health services (i.e. Column (1) and (3) in the table 16).

Table 16: Regressing inpatients, hospital revenue and outpatients service on the number of enrollees whose premium is covered by rural medical assistance fund

	(1) inpatients	(2) revenue	(3) outpatients
h_doctors	33774.7	274469.9	270625.2*
	(27410.7)	(424802.7)	(146407.1)
rural_insurance_assistance	-5522.3	-95030.4	25772.9
	(4277.3)	(66287.8)	(23616.3)
urban_insurance_assistance	10712.1	-169048.1	-56382.9
	(6659.1)	(103200.1)	(38449.2)
age65	-597643.2	18719144.8**	3309775.0
	(605610.1)	(9385575.8)	(3495499.2)
h_beds	228209.6***	567773.8**	
	(18169.1)	(281579.6)	
bhi	1753.6	167192.8***	14911.2
	(3091.9)	(47917.0)	(17807.3)
disposable_income	5922.9**	181391.8***	101808.2***
	(2585.2)	(40064.2)	(12476.5)
cms	1358.7*	-56041.2***	-5913.7
	(766.1)	(11872.1)	(4061.4)
urbmi	385.4	-24012.1***	-1105.3
	(415.4)	(6437.7)	(2350.6)
constant	-279902.3***	-2741523.2**	1752125.6***
	(78675.2)	(1219287.0)	(454002.2)
N	123	123	123
adj. R^2	0.916	0.750	0.767

Standard errors in parentheses

* p<.1, ** p<0.05, *** p<0.01

For poor households who are subsidized with the premium of health insurance, they are still required to pay the deductible as well as copayment for healthcare services. For rural health insurance, the copayment can be as high as 50% for inpatient services. Deductible for outpatient service could be even higher. Many poor households may not visit a doctor since they may not afford the deductible and copayment. In this case, the medical assistance fund is not spent efficiently since the health insurance premium is useful for improving people's welfare only when people utilize the healthcare services. Poor households may benefit instead by directly reimbursed by rural medical assistance fund.

Column (2) in table 16 suggests that rural health insurance and urban resident medical insurance (i.e. the coefficient *cms* and *urbmi*) have a negative impact on hospital revenue since both of these plans are more restrict on the drugs and treatment procedure and hospital revenue is more likely to be less for an enrollee under these two plans. However, the number of poor households whose insurance premium is covered by the medical assistance fund is not associated with the hospital revenue. It may also suggest the utilization of health services for those poor households is not very significant.

4.3. Inefficiency in the under supply of infrastructure in mental health care

Because mental health care are not the priority issue for both MCA and MOH, there is deficiency in the supply of infrastructure in mental health care. In this subsection, we show that supply side reason is the major constraint for mental health care. In other words, the infrastructure to treat patients with mental diseases is underdeveloped. We use two groups of hospitals to test this hypothesis. One group is those mental hospitals managed by MOH and the other group is mental hospitals managed by MCA.

This is a provincial level data analysis. The time span for the data is between 2007 and 2012 (i.e. 6 years). Data sources: China Health Statistical Yearbook and China Civil Affairs Statistical Yearbook, Various years.

4.3.1. Variables :

The dependent variable is defined as the volume of inpatient services in two types of mental hospitals: MOH (ministry of health) and MCA (ministry of civil affairs) in a province.

The major independent variable is the number of beds in two types of hospitals in a province.

The control variables such as income, education, manufacturing sector as well as insurance measure the demand for mental health care. People residing in a more economic developed region, in a better educated region, in a region with a larger manufacturing sector and more generous insurance coverage are likely to demand more mental health care from the literature.

People residing in a more economically developed and better educated region are more likely to be informed about the mental diseases. A province with a larger manufacturing sector is more likely to have more workers working in a mass production process and suffer mental diseases (thinking about migrant workers). People residing in a province with better insurance plan will be covered for the fees charged by the mental hospitals.

All variables are weighted by population and the description of variables is shown in table 17 and 18.

Table 17: Descriptions of variables:

Variable name	**Variable description**
mental_inpatients	Inpatient visits in MOH hospital (per million people)
mca_inpatient	Inpatient visits in MCA hospital (per million people)
mental_beds	Bed for patients with mental diseases in MOH hospitals (per million people)
mca_mental_beds	Bed for patients with mental diseases in MOH hospitals (per million people)
education_ratio	Education(share of people with tertiary education or above)
industry_ratio	Industry (The share of manufacturing sector in total employment)
disposable_income	Average Disposable income in the province (1,000 RMB)
urban_insurance	Average health expenditure for urban social health insurance(1,000 RMB)

Table 18: Summary of variables

	count	mean	sd	min	max
mental_inpatients	182	769.0363	379.067	119.354	2011.715
mental_beds	182	158.641	116.999	8.1269	596.239
mca_inpatients	172	33.68	23.78	0.01	149.620
mca_mental_beds	172	48.71	30.77	6.64	127.22
education_ratio	182	9.457	5.821	3.064	37.350
urban_insurance	182	1.382	.585	.377	3.909
industry_ratio	182	24.442	10.208	2.625	50.549
disposable_income	182	17.806	5.785	10.012	40.188

Table 19 below shows the regression results. Column (1) and (3) is the basic correlation between the volume of services and the number of beds. Some demand side conditions are added in column (2) and (4). It turns out that in column (2) and (4), non-of these demand side conditions such as income, urbanization, the size of manufacture sector, education level is significant. Also, the magnitude of coefficient for the number of beds is hardly affected by adding these demand side variables. The regression results in this table, therefore, show that the supply side constraint is major reason for under treatment of patients with mental diseases.

Table 19: Regression results showing the determinants of utlization of mental healthcare

	(1) mca_inpatients	(2) mental_inpatients	(3) mca_inpatients	(4) mental_inpatients
mca_mental_beds	0.499***		0.484***	
	(0.0913)		(0.0932)	
mental_beds		5.301***		6.918***
		(0.795)		(0.873)
education_ratio		4.633	0.306	
		(6.346)	(0.617)	
urban_insurance		62.37	3.386	
		(68.18)	(5.706)	
industry_ratio		5.234	0.889	
		(5.671)	(0.918)	
disposable_income		9.381	-0.485	
		(6.703)	(0.508)	
constant	9.367**	-496.9**	-11.24	-328.4**
	(4.449)	(185.9)	(24.81)	(138.5)
N	172	182	172	182
adj. R^2	0.132	0.763	0.125	0.695

Standard errors in parentheses

* p<.1, ** p<0.05, *** p<0.01

5. Conclusion

This paper develops a formal model to study the effect of logrolling on policy making. We compare the policy outcome under logrolling with policy outcome under other decision-making rules. We find that, policies under logrolling tend to be inefficiently high and policies excluded from logrolling tend to be inefficiently low. Besides, the first-mover advantage we observed in the case of bargaining no longer exists in the case of logrolling. We provide empirical evidence by studying the logrolling between Ministry of Civil Affairs and Ministry of Health. The preliminary results show that there is inefficiency due to policy overreaching in "Dibao" and rural health insurance, and there is insufficient input in infrastructure building in mental health care.

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6. Appendix

6.1. Kuhn-Tucker conditions when decision is made by a benevolent social planner

The Lagrangian is

$$L(x,y) = 2 - 2x - 2y + \sqrt{x} + \sqrt{y} + \lambda(1 - x - y) \tag{28}$$

and the Kuhn-Tucker conditions for this Lagrangian are:

$$\frac{\partial L}{\partial x} = -2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0, \quad x \geq 0, \ and \ x(-2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda) = 0 \tag{29}$$

$$\frac{\partial L}{\partial y} = -2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda \leq 0, \quad y \geq 0, \quad and \ y(-2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda) = 0 \tag{30}$$

$$\lambda \geq 0, \quad x + y - 1 \leq 0, \quad and \quad \lambda(x + y - 1) = 0 \tag{31}$$

We can find solutions of these conditions as follows:

- $-2 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow x > 0$;
- $-2 + \frac{1}{2}y^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow y > 0$;
- hence $-2 + \frac{1}{2}x^{-\frac{1}{2}} = \lambda = -2 + \frac{1}{2}y^{-\frac{1}{2}} \Rightarrow x = y$;
- if $\lambda > 0$ then $x + y - 1 = 0$ which implies $x = y = \frac{1}{2}$. Then $\lambda = -2 + \frac{\sqrt{2}}{2} < 0$, contradicting $\lambda > 0$;
- hence $\lambda = 0$, which in turn implies $x = y = \frac{1}{16}$.

6.2. Kuhn-Tucker conditions when decision is made by one interest group

The Kuhn-Tucker conditions are:

$$-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0, \quad x \geq 0, \ and \ x\left(-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda\right) = 0 \tag{32}$$

$$\lambda \geq 0, \quad x - 1 \leq 0, \quad and \quad \lambda(x - 1) = 0 \tag{33}$$

We can find solutions of these conditions as follows:

- $-1 + \frac{1}{2}x^{-\frac{1}{2}} - \lambda \leq 0 \Rightarrow x > 0$;
- if $\lambda > 0$ then $x - 1 = 0$ which implies $\lambda = -\frac{1}{2} < 0$, contradicting $\lambda > 0$;
- hence $\lambda = 0$, which in turn implies $x = \frac{1}{4}$.

6.3. Proof of porposition 1

In this appendix, we will prove porposition 1. We can work out this game from backward.

6.3.1. STEP 4:

In the last decision step, α chooses between to accept the policy proposed by β and the default policy. We should distinguish two different set of subgames, whether in step 2 β rejected or accepted α's proposal. If $c^\beta = N$, then α's choice is irrelevant as both players will get the default outcome. If $c^\beta = Y$, then $c^\alpha = Y$ if and only if

$$U^\alpha\left(Y|h^3 = (x, Y, y)\right) \geq U^D \iff 1 - x - y + \sqrt{x} \geq \theta \iff y \leq 1 + \sqrt{x} - x - \theta \quad (34)$$

The condition defined by inequality (34) is α's "incentive logrolling constraint".

Hence, we get the following sequential best reply for α:

$$SBR^\alpha(h^3) = \begin{cases} \in \{Y, N\}, if\ h^3 = (x, N, y) \\ N,\ if\ h^3 = (x, Y, y)\ s.t.\ y \geq 1 + \sqrt{x} - x - \theta \\ Y,\ if\ h^3 = (x, Y, y)\ s.t.\ y \leq 1 + \sqrt{x} - x - \theta \end{cases} . \quad (35)$$

6.3.2. STEP 3:

In step 3, β will choose $y^{\alpha L}$ to maximize his utility subject to SBR^α. Again, we should distinguish two different set of subgames, i.e. whether in step 2 β rejected or accepted α's proposal. If $c^\beta = N$, then β's choice is irrelevant as both players will anyway get the default outcome. Hence

1. if $h^2 = (x, N)$, then $SBR^\beta(h^2) \in [0, 1]$;
2. if $h^2 = (x, Y)$, then $SBR^\beta(h^2) \in argmax_y U^\beta\left(y|h^2 = (x, Y), SBR^\alpha(h^3)\right)$.

Note that $y^{\alpha L} > 1 + \sqrt{x} - x - \theta \Longrightarrow U^\beta\left(y^{\alpha L}|h^2 = (x, S), SBR^\alpha(h^3)\right) = \theta$. Otherwise, we have the following maximization problem:

$$max_y\left(1 - x - y + \sqrt{y}\right) \quad (36)$$

$$s.t.\ y \leq 1 + \sqrt{x} - x - \theta \quad \text{(IL constraint)}$$

$$y \leq 1 - x \quad \text{(Resource constraint)}$$

$$y \in [0, 1] \quad (37)$$

First note that the IL constraint is more or restrictive than the resource constraint according to the possible values of θ. In particular

$$1 + \sqrt{x} - x - \theta \leq 1 - x \Leftrightarrow x \leq \theta^2.$$

Hence we distinguish two maximization problems:

1. when $x \leq \theta^2$,

$$max_y\left(1 - x - y + \sqrt{y}\right) \quad (38)$$

$$s.t.\ y \leq 1 + \sqrt{x} - x - \theta \quad \text{(IL constraint)}$$

$$y \geq 0 \quad (39)$$

2. when $x \geq \theta^2$,

$$max_y \left(1 - x - y + \sqrt{y}\right) \tag{40}$$

$$s.t.\ y \leq 1 - x \qquad \text{(Resource constraint)}$$

$$y \geq 0 \tag{41}$$

Moreover, β objective function has an unconstrained maximum in $y = \frac{1}{4} \gtrless (1 + \sqrt{x} - x - \theta)$ or $(1 - x)$ depending on x and θ. In particular

$$\frac{1}{4} \leq 1+\sqrt{x}-x-\theta \Leftrightarrow \sqrt{x}-x \geq \theta-\frac{3}{4} \Leftrightarrow \begin{cases} x \in [0,1] & \text{if } \theta \leq \frac{3}{4} \\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] & \text{if } \theta \geq \frac{3}{4} \end{cases}$$

and

$$\frac{1}{4} \leq 1 - x \Leftrightarrow x \in \left[0, \frac{3}{4}\right].$$

Hence

1. when $x \leq \theta^2 \ \ \& \ \ \theta \in \left[0, \frac{3}{4}\right]$,

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in [0,1] \\ \in [0,1] & if\ \ h^2 = (x, N)\ \&\ x \in [0,1]\,; \end{cases}$$

2. when $x \leq \theta^2 \ \ \& \ \ \theta \in \left[\frac{3}{4}, 1\right]$,

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ \ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \\ 1 + \sqrt{x} - x - \theta & if\ h^2 = (x, S)\ \& \\ & x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \cup \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right] \\ \in [0,1] & if\ \ h^2 = (x, N)\ \&\ x \in [0,1]\,; \end{cases}$$

3. when $x \geq \theta^2$,

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[0, \frac{3}{4}\right] \\ 1 - x & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \\ \in [0,1] & if\ \ h^2 = (x, N). \end{cases}$$

In case 2 when $x \leq \theta^2 \ \ \& \ \ \theta \in \left[\frac{3}{4}, 1\right]$, consider

$$x \in \left[0, \theta^2\right] \cap \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \Leftrightarrow$$

$$\Leftrightarrow x \in \begin{cases} \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \theta^2\right] & \theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right] \\ \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] & \theta \in \left[\frac{\sqrt{3}}{2}, 1\right] \end{cases}$$

since

$$\frac{5}{4} - \theta - \sqrt{1-\theta} < 0 \Leftrightarrow \theta \in \varnothing$$

$$\frac{5}{4} - \theta + \sqrt{1-\theta} \geq \theta^2 \Leftrightarrow \theta \in \left[0, \frac{\sqrt{3}}{2}\right]$$

Hence we can summarize the sequential best response of β when $y^{\alpha L} \leq 1 + \sqrt{x} - x - \theta$ as follows:

1. $\theta \in \left[0, \frac{3}{4}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{3}{4}\right] \\ 1 - x & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \\ \in [0,1] & if\ \ h^2 = (x,N). \end{cases}$$

2. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} 1 + \sqrt{x} - x - \theta & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{1}{4} & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{3}{4}\right] \\ 1 - x & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \\ \in [0,1] & if\ \ h^2 = (x,N). \end{cases}$$

3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^{\beta}(h^2) = \begin{cases} 1 + \sqrt{x} - x - \theta & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{1}{4} & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \\ 1 + \sqrt{x} - x - \theta & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right] \\ \in [0,1] & if\ \ h^2 = (x,N). \end{cases}$$

Following these rules, the maximum utility would be:

1. $\theta \in \left[0, \frac{3}{4}\right]$

$$U^{\beta}\left(SBR^{\beta}(h^2)\right) = \begin{cases} \frac{5}{4} - x & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{3}{4}\right] \\ \sqrt{1-x} & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \\ \theta & if\ \ h^2 = (x,N). \end{cases}$$

2. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$U^{\beta}\left(SBR^{\beta}(h^2)\right) = \begin{cases} \theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{5}{4} - x & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{3}{4}\right] \\ \sqrt{1-x} & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \\ \theta & if\ \ h^2 = (x,N). \end{cases}$$

3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$U^{\beta}\left(SBR^{\beta}(h^2)\right) = \begin{cases} \theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} & if\ h^2 = (x,S)\ \&\ x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{5}{4} - x & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \\ \theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} & if\ h^2 = (x,S)\ \&\ x \in \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right] \\ \theta & if\ \ h^2 = (x,N). \end{cases}$$

Otherwise, if $y^{\alpha L} > 1+\sqrt{x}-x-\theta$, then β can guarantee himself $U^{\beta}\left(y^{\alpha L} | h^2 = (x,S), SBR^{\alpha}(h^3)\right) = \theta$, hence we should check that

$$U^{\beta}\left(SBR^{\beta}(h^2)\right) \geq \theta$$

i.e.

1. when $\theta \in \left[0, \frac{3}{4}\right]$ & $x \in \left[0, \frac{3}{4}\right]$

$$\frac{5}{4} - x \geq \theta \Leftrightarrow x \leq \frac{5}{4} - \theta$$

2. when $\theta \in \left[0, \frac{3}{4}\right]$ & $x \in \left[\frac{3}{4}, 1\right]$

$$\sqrt{1-x} \geq \theta \Leftrightarrow 1 - x \geq \theta^2 \Leftrightarrow x \leq 1 - \theta^2$$

3. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$ & $x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right]$

$$\theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} \geq \theta \Leftrightarrow \sqrt{1+\sqrt{x}-x-\theta} \geq \sqrt{x} \Leftrightarrow 1 + \sqrt{x} - x - \theta \geq x \Leftrightarrow$$

$$\Leftrightarrow 2x - \sqrt{x} \leq 1 - \theta \Leftrightarrow x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9-8\theta}\right]$$

since $2x+\theta-1-\sqrt{x} = 0$, Solution is: $\left\{-\frac{1}{2}\theta - \frac{1}{8}\sqrt{-8\theta+9} + \frac{5}{8}, -\frac{1}{2}\theta + \frac{1}{8}\sqrt{-8\theta+9} + \frac{5}{8}\right\}$

4. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$ & $x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{3}{4}\right]$

$$\frac{5}{4} - x \geq \theta \Leftrightarrow x \leq \frac{5}{4} - \theta$$

5. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$ & $x \in \left[\frac{3}{4}, 1\right]$

$$\sqrt{1-x} \geq \theta \Leftrightarrow 1 - x \geq \theta^2 \Leftrightarrow x \leq 1 - \theta^2$$

6. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$ & $x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right]$

$$\theta - \sqrt{x} + \sqrt{1+\sqrt{x}-x-\theta} \geq \theta \Leftrightarrow \sqrt{1+\sqrt{x}-x-\theta} \geq \sqrt{x} \Leftrightarrow 1 + \sqrt{x} - x - \theta \geq x \Leftrightarrow$$

$$\Leftrightarrow 2x - \sqrt{x} \leq 1 - \theta \Leftrightarrow x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9-8\theta}\right]$$

since $2x+\theta-1-\sqrt{x} = 0$, Solution is: $\left\{-\frac{1}{2}\theta - \frac{1}{8}\sqrt{-8\theta+9} + \frac{5}{8}, -\frac{1}{2}\theta + \frac{1}{8}\sqrt{-8\theta+9} + \frac{5}{8}\right\}$

7. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$ & $x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right]$

$$\frac{5}{4} - x \geq \theta \Leftrightarrow x \leq \frac{5}{4} - \theta$$

8. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$ & $x \in \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right]$

$$\theta - \sqrt{x} + \sqrt{1 + \sqrt{x} - x - \theta} \geq \theta \Leftrightarrow \sqrt{1 + \sqrt{x} - x - \theta} \geq \sqrt{x} \Leftrightarrow 1 + \sqrt{x} - x - \theta \geq x \Leftrightarrow$$

$$\Leftrightarrow 2x - \sqrt{x} \leq 1 - \theta \Leftrightarrow x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9 - 8\theta}\right]$$

since $2x + \theta - 1 - \sqrt{x} = 0$, Solution is: $\left\{-\frac{1}{2}\theta - \frac{1}{8}\sqrt{-8\theta + 9} + \frac{5}{8}, -\frac{1}{2}\theta + \frac{1}{8}\sqrt{-8\theta + 9} + \frac{5}{8}\right\}$.

Hence the sequential best response of β is:

1. $\theta \in \left[0, \frac{3}{4}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[0, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] \\ 1 - x & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \cap \left[0, 1 - \theta^2\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

2. $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} & if\ h^2 = (x, S)\ \& \\ 1 + \sqrt{x} - x - \theta & x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9} - \right. \\ \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] \\ 1 - x & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{3}{4}, 1\right] \cap \left[0, 1 - \theta^2\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

3. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^{\beta}(h^2) = \begin{cases} & if\ h^2 = (x, S)\ \& \\ 1 + \sqrt{x} - x - \theta & x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9} - \right. \\ \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \cap \left[0, \frac{5}{4}\right. \\ & if\ h^2 = (x, S)\ \& \\ 1 + \sqrt{x} - x - \theta & x \in \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9} - \right. \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

This sequential best replies can be simplified since:

1. when $\theta \in \left[0, \frac{3}{4}\right]$

$$x \in \left[0, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \begin{cases} \left[0, \frac{3}{4}\right] & \theta \in \left[0, \frac{1}{2}\right] \\ \left[0, \frac{5}{4} - \theta\right] & \theta \in \left[\frac{1}{2}, \frac{3}{4}\right] \end{cases}$$

2. when $\theta \in \left[0, \frac{3}{4}\right]$

$$x \in \left[\frac{3}{4}, 1\right] \cap \left[0, 1 - \theta^2\right] = \begin{cases} \left[\frac{3}{4}, 1 - \theta^2\right] & \theta \in \left[0, \frac{1}{2}\right] \\ \varnothing & \theta \in \left[\frac{1}{2}, \frac{3}{4}\right] \end{cases}$$

3. when $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9-8\theta}\right] =$$

$$= \begin{cases} \varnothing & \theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \\ \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] & \theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \end{cases}$$

since $5 - 4x + \sqrt{9-8x} - 8\sqrt{1-x} = 0$, Solution is: $0, \frac{3}{2}\sqrt{5} - \frac{5}{2}$

4. when $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right]$$

5. when $\theta \in \left[\frac{3}{4}, \frac{\sqrt{3}}{2}\right]$

$$x \in \left[\frac{3}{4}, 1\right] \cap \left[0, 1 - \theta^2\right] = \varnothing$$

6. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$x \in \left[0, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9-8\theta}\right] =$$

$$= \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right]$$

7. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta + \sqrt{1-\theta}\right] \cap \left[0, \frac{5}{4} - \theta\right] =$$

$$= \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right]$$

8. when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$x \in \left[\frac{5}{4} - \theta + \sqrt{1-\theta}, 1\right] \cap \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{8} - \frac{1}{2}\theta + \frac{1}{8}\sqrt{9-8\theta}\right] = \begin{cases} \varnothing & \theta \in \left[\frac{\sqrt{3}}{2}, 1\right) \\ \left\{\frac{1}{4}\right\} & \theta = 1. \end{cases}$$

Hence the sequential best response of β is:

1. $\theta \in \left[0, \frac{1}{2}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[0, \frac{3}{4}\right] \\ 1 - x & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{3}{4}, 1 - \theta^2\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

2. $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[0, \frac{5}{4} - \theta\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

3. $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

4. $\theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right]$

$$SBR^{\beta}(h^2) = \begin{cases} 1 + \sqrt{x} - x - \theta & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise \end{cases}$$

5. $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right]$

$$SBR^{\beta}(h^2) = \begin{cases} 1 + \sqrt{x} - x - \theta & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \frac{1}{4} & if\ h^2 = (x, S)\ \&\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \in [0, 1] & if\ \ h^2 = (x, N) \\ \in [1 + \sqrt{x} - x - \theta, 1] & otherwise. \end{cases}$$

6.3.3. STEP 2

In step 2, β will choose whether to support α or not. In particular, $c^{\beta} = Y$ if and only if

$$U^{\beta}(Y|x, SBR^{\beta}(h^2), SBR^{\alpha}(h^3)) \geq U^{S} \Leftrightarrow 1 - x - SBR^{\beta}(h^2) + \sqrt{SBR^{\beta}(h^2)} \geq \theta \Leftrightarrow \quad (42)$$

$$\Leftrightarrow x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2).$$

Hence, again we need to distinguish five cases for θ:

1. Suppose $\theta \in \left[0, \frac{1}{2}\right]$, then β will choose $c^{\beta}(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2) \Leftrightarrow$$

$$\Leftrightarrow x \in \begin{cases} \left[0, \frac{5}{4} - \theta\right] & if\ x \in \left[0, \frac{3}{4}\right] \\ \left[0, x - \theta + \sqrt{1 - x}\right] & if\ x \in \left[\frac{3}{4}, 1 - \theta^2\right] \end{cases}$$

   i.e. if and only if

$$x \in \begin{cases} \left[0, \frac{3}{4}\right] \cap \left[0, \frac{5}{4} - \theta\right] = \left[0, \frac{3}{4}\right] \\ \left[\frac{3}{4}, 1 - \theta^2\right] \cap \left[0, x - \theta + \sqrt{1 - x}\right] = \left[\frac{3}{4}, 1 - \theta^2\right]. \end{cases}$$

   Therefore when $\theta \in \left[0, \frac{1}{2}\right]$

$$c^{\beta}(x) = \begin{cases} Y & if\ \ x \in \left[0, 1 - \theta^2\right] \\ N & otherwise. \end{cases}$$

2. suppose $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$, then β will choose $c^{\beta}(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2) \Leftrightarrow$$

$$\Leftrightarrow x \in \left\{ \left[0, \frac{5}{4} - \theta\right] \quad if\ x \in \left[0, \frac{5}{4} - \theta\right] \right.$$

   i.e. if and only if

$$x \in \left[0, \frac{5}{4} - \theta\right]$$

   Therefore when $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$

$$c^{\beta}(x) = \begin{cases} Y & if\ \ x \in \left[0, \frac{5}{4} - \theta\right] \\ N & otherwise. \end{cases}$$

3. suppose $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \simeq [0.75, 0.854]$, then β will choose $c^{\beta}(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2) \Leftrightarrow$$

$$\Leftrightarrow x \in \left\{ \left[0, \frac{5}{4} - \theta\right] \quad if\ x \in \left[\frac{5}{4} - \theta - \sqrt{1 - \theta}, \frac{5}{4} - \theta\right] \right.$$

i.e. if and only if

$$x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right]$$

Therefore when $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \simeq [0.75, 0.854]$

$$c^{\beta}(x) = \begin{cases} Y & if \quad x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ N & otherwise. \end{cases}$$

4. suppose $\theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \simeq [0.854, 0.866]$, then β will choose $c^{\beta}(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2) \Leftrightarrow$$

$$\Leftrightarrow x \in \begin{cases} \left[0, \sqrt{\sqrt{x} - x}\right] & if\ x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \left[0, \frac{5}{4} - \theta\right] & if\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \end{cases}$$

i.e. if and only if

$$x \in \begin{cases} \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \end{cases} = \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta\right]$$

Therefore when $\theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \simeq [0.854, 0.866]$

$$c^{\beta}(x) = \begin{cases} Y & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta\right] \\ N & otherwise. \end{cases}$$

5. suppose $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right] \simeq [0.866, 1]$, then β will choose $c^{\beta}(x) = Y$ if and only if

$$x \leq 1 - \theta + \sqrt{SBR^{\beta}(h^2)} - SBR^{\beta}(h^2) \Leftrightarrow$$

$$\Leftrightarrow x \in \begin{cases} \left[0, \sqrt{\sqrt{x} - x}\right] & if\ x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \left[0, \frac{5}{4} - \theta\right] & if\ x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \end{cases}$$

i.e. if and only if

$$x \in \begin{cases} \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \end{cases} = \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta\right]$$

Therefore when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right] \simeq [0.866, 1]$

$$c^{\beta}(x) = \begin{cases} Y & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta\right] \\ N & otherwise. \end{cases}$$

6.3.4. STEP 1

Moving backward, in step 1, α will choose $x \in [0,1]$ to maximize

$$U^{\alpha}(x|SBR^{\beta}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$$

that we will denote by $U^{\alpha}(x|\theta)$ since it depends on the possible values of θ

Case 1: $\theta \in \left[0, \frac{1}{2}\right]$

$$U^{\alpha}(x|\theta) = \begin{cases} 1 - \underbrace{\frac{1}{4}}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & if \quad x \in \left[0, \frac{3}{4}\right] \\ 1 - \underbrace{(1-x)}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \sqrt{x} & if \quad x \in \left[\frac{3}{4}, 1-\theta^2\right] \\ \theta & otherwise. \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^{\alpha}(x|\theta) = \begin{cases} 1 & if \quad x = \frac{1}{4} \\ \sqrt{1-\theta^2} & if \quad x = 1-\theta^2 \\ \theta & otherwise. \end{cases}$$

Thus when $\theta \in \left[0, \frac{1}{2}\right]$, $U^{\alpha}(x, SBR^{\alpha}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the defaul utility θ

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

Case 2: $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$

$$U^{\alpha}(x|\theta) = \begin{cases} 1 - \underbrace{\frac{1}{4}}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & if \quad x \in \left[0, \frac{5}{4} - \theta\right] \\ \theta & otherwise. \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^{\alpha}(x|\theta) = \begin{cases} 1 & if \quad x = \frac{1}{4} \\ \theta & otherwise. \end{cases}$$

Thus when $\theta \in \left[\frac{1}{2}, \frac{3}{4}\right]$, $U^{\alpha}(x, SBR^{\alpha}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the defaul utility θ

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

Case 3: $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \simeq [0.75, 0.854]$

$$U^{\alpha}(x|\theta) = \begin{cases} 1 - \underbrace{\frac{1}{4}}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & if \quad x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \theta & otherwise \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^{\alpha}(x|\theta) = \begin{cases} 1 & if \quad x = \frac{1}{4} \\ \theta & otherwise. \end{cases}$$

Thus when $\theta \in \left[\frac{3}{4}, \frac{3\sqrt{5}}{2} - \frac{5}{2}\right] \simeq [0.75, 0.854]$, $U^{\alpha}(x, SBR^{\alpha}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the defaul utility θ

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

Case 4: $\theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \simeq [0.854, 0.866]$

$$U^{\alpha}(x|\theta) = \begin{cases} 1 - \underbrace{\left(1 + \sqrt{x} - x - \theta\right)}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \theta & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ 1 - \underbrace{\frac{1}{4}}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & if \quad x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \theta & otherwise \end{cases}$$

Hence

$$\max_{x \in [0,1]} U^{\alpha}(x|\theta) = \begin{cases} \theta & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ 1 & if \quad x = \frac{1}{4} \\ \theta & otherwise. \end{cases}$$

Thus when $\theta \in \left[\frac{3\sqrt{5}}{2} - \frac{5}{2}, \frac{\sqrt{3}}{2}\right] \simeq [0.854, 0.866]$, $U^{\alpha}(x, SBR^{\alpha}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the defaul utility θ.

Case 5: $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right] \simeq [0.866, 1]$

$$U^{\alpha}(x|\theta) = \begin{cases} 1 - \underbrace{\left(1 + \sqrt{x} - x - \theta\right)}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \theta & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9 - 8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ 1 - \underbrace{\frac{1}{4}}_{-SBR^{\beta}(h^2)} - x + \sqrt{x} = \frac{3}{4} - x + \sqrt{x} & if \quad x \in \left[\frac{5}{4} - \theta - \sqrt{1-\theta}, \frac{5}{4} - \theta\right] \\ \theta & otherwise \end{cases}$$

Hence

$$\max_{x\in[0,1]} U^{\alpha}(x|\theta) = \begin{cases} \theta & if \quad x \in \left[\frac{5}{8} - \frac{1}{2}\theta - \frac{1}{8}\sqrt{9-8\theta}, \frac{5}{4} - \theta - \sqrt{1-\theta}\right] \\ 1 & if \quad x = \frac{1}{4} \\ \theta & otherwise. \end{cases}$$

Thus when $\theta \in \left[\frac{\sqrt{3}}{2}, 1\right] \simeq [0.866, 1]$, $U^{\alpha}(x, SBR^{\alpha}(x), SBR^{\beta}(h^2), SBR^{\alpha}(h^3))$ is maximized for $x = \frac{1}{4}$ and the solution does not depend on the defaul utility θ

$$\left(h^{\alpha L}, x^{\alpha L}, y^{\alpha L}\right) = \left(h^{\beta L}, x^{\beta L}, y^{\beta L}\right) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$